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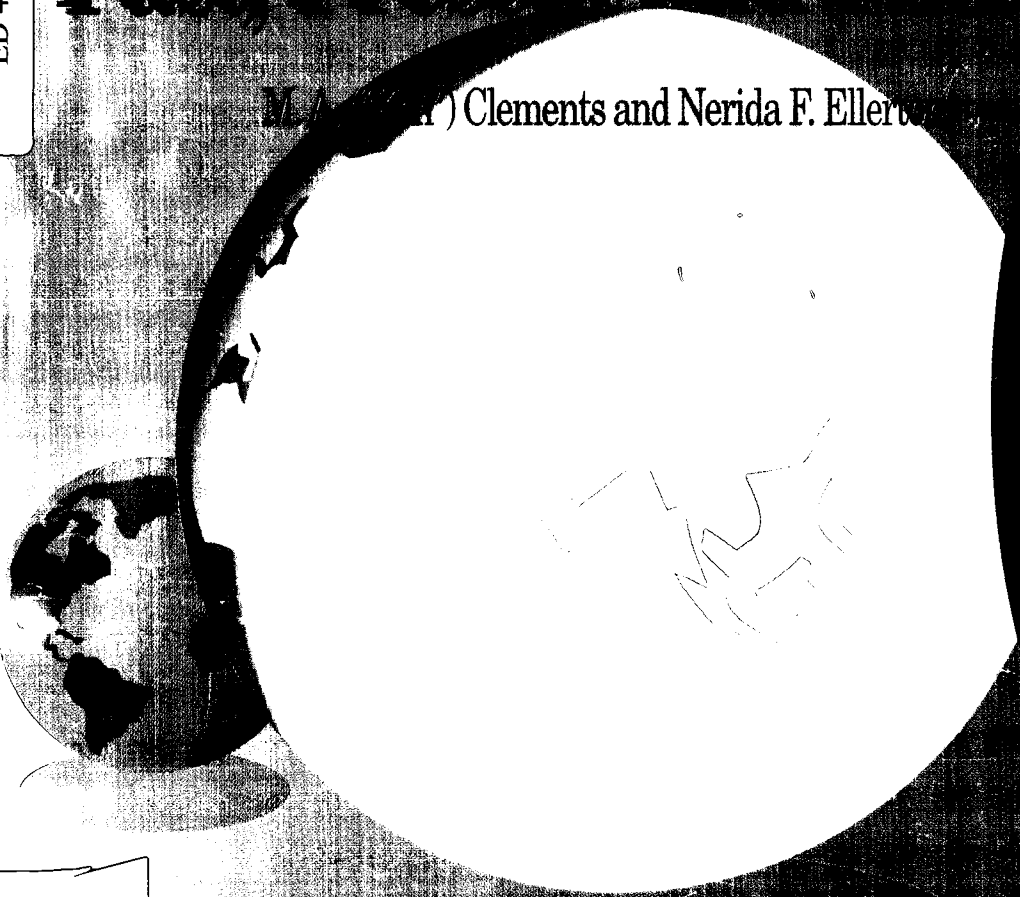
ABSTRACT

As countries in the vast Asia-Pacific region seek to re-engineer their education systems in order to cope with the rapid pace of economic and social change, an area of the school curriculum that is attracting increasing special attention is that of mathematics education. The work in this book identifies key issues regarding mathematics education in different parts of the world and considers whether these issues have implications for the future of mathematics education in the Asia-Pacific region. The assumptions, methods, and values that underpin contemporary mathematics education research are questioned, and important gaps in contemporary mathematics literature that need to be addressed are identified. (Contains 587 references.) (ASK)

Mathematics Education Research

Past, Present and Future

M. A. (Ken) Clements and Nerida F. Ellerton



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M. A. ("Ken") Clements and Nerida F. Ellerton

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Foreword

This is an important book on an important subject.

As countries in the vast Asia-Pacific region seek to re-engineer their education systems to cope with the rapid pace of economic and social change, an area of the school curriculum which is attracting special, increasing attention is that of mathematics education. This interest is not surprising since mathematics is viewed by nations throughout the region as being an essential key to gaining access to the benefits of technological change, with all the implications this has for enabling countries to reap the benefits of economic development and modernisation.

Thus mathematics education is an integral part of the curriculum in all schools of the region, all nations investing vast resources in support of the teaching of mathematics.

In this ground-breaking and controversial work, Professors Clements and Ellerton identify key issues regarding mathematics education, in different parts of the world, and then consider whether these have implications for the future of mathematics education in the Asia-Pacific region. The authors question many of the assumptions, methods and values which underpin contemporary mathematics education research, and identify important gaps in the contemporary mathematics literature which need to be addressed.

In questioning the agenda and methods of much of contemporary mathematics education research, the authors put this important field of research under the microscope in order to help re-conceptualise the policies, practices and research associated with mathematics education. In so doing they suggest a new epistemological framework for mathematics education research.

Clements and Ellerton evaluate many of the taken-for-granted assumptions currently held by the international mathematics education community and find them wanting.

This book has much to teach us not only about mathematics education but much else besides. It is to be hoped that policy makers and practitioners alike will learn from the mistakes and weaknesses that have occurred elsewhere in the world regarding mathematics education research, and that they will take steps to ensure that these are not repeated in this region. The book deserves to be widely read, particularly in the Asia-Pacific region, where nations are striving to improve the quality, effectiveness and relevance of mathematics education for all students.

Rupert Maclean
Chief of the Asia-Pacific Centre of Educational Innovation for Development (ACEID)

UNESCO, Bangkok
December, 1996

Preface

The entry for "Asia-Pacific region" in the Subject Index to this book is longer than the entry for "United States of America." That is probably the way it should be, given that the main focus of the book is mathematics education research in the Asia-Pacific region. However, since this is the first book ever to be written with that focus, we continually found ourselves torn between wanting to report the theories and findings of Western education researchers and the need to be of maximum benefit to present and future mathematics educators in the region. The difficult issue of whether "Western" theories and findings should be de-emphasised because they represent an essentially Eurocentric view of education, in general, and of mathematics education in particular, was everpresent in our minds.

For example, despite the reverence which Continental, and other, education researchers have maintained towards the writings of Piaget and Vygotsky, we are not convinced that anything other than the essence of these writings has a great deal of relevance in education settings in many of the nations which make up the Asia-Pacific region. After all, it was on the basis of Piagetian stage theories that researchers, in the period 1950–1980, carried out and reported investigations which purported to show that many children in Asia-Pacific nations—such as students in Papua New Guinea Community Schools, and Aboriginal Australian children—thought in "prelogical" ways and were often three or four years "behind" Western children from the perspective of cognitive development. Such conclusions can now be seen as degrading and highly value-laden.

Similarly, we are not convinced that the Continental *didactique* research, in which mathematics classroom discourse patterns are studied and interpreted in terms of codes of teacher, student and societal behaviours, relationships and priorities, has much relevance in the nations of the Asia-Pacific region where linguistic and cultural patterns are typically very different from those in Europe.

Also, given the current questioning in the West of the legitimacy of Fisherian statistical significance testing (SST), one has to query whether Asia-Pacific researchers derived any benefits at all from using SST techniques. Arguably, they have lost a great deal by mimicking faulty research techniques which were assumed to be correct simply because they were popular in "advanced" Western nations.

In Chapter 1 we point out that the Asia-Pacific region—as we have defined it—covers about half of the world's population, that mathematics is an integral part of the curriculum in all schools of the region, and that many billions of "dollars" are invested in the resources needed to support the teaching of mathematics each year by the nations. Clearly, mathematics education research is (or at least should be) of fundamental importance to the region. One purpose of this book is to identify key issues which have concerned mathematics educators researchers around the world, and to consider whether these have any bearing on the future of mathematics education in the Asia-Pacific region. A second purpose is to begin to identify

underlying factors which, although they influence mathematics education practices in all or part of the region, have *not* been adequately addressed in mainstream international mathematics education literatures. A third purpose is to stimulate mathematics educators around the world to reflect aggressively on the assumptions, methods and values inherent in contemporary mathematics education research.

We would like to thank Rupert Maclean, Chief of the Asia-Pacific Centre of Educational Innovation for Development (UNESCO, Bangkok), for inviting us to write the book and for encouraging us throughout its gestation period.

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1 Towards a Balanced Perspective on Contemporary Mathematics Education Research

Introduction

This book has two main purposes: the first is to summarise and critique international trends in mathematics education research; the second is to develop a set of recommendations concerning the role and potential of mathematics education research. There will be a special emphasis on research into issues associated with mathematics curricula, teaching, and learning in the Asia-Pacific region—which will be taken to include nations within East Asia, Southeast Asia, Micronesia, Oceania, Polynesia, Australia, New Zealand, Papua New Guinea, and certain Southern Asian nations within the "Indian sub-continent."[1]

The book does not give detailed advice on how to go about doing mathematics education research. Nor does it include detailed reviews of the main literatures in the field of mathematics education, for these are already available in two comprehensive edited collections (Bishop, Clements, Keitel, Kilpatrick, & Laborde, 1996; Grouws,1992).

The perspectives presented in this book are by no means traditional. Despite the best intentions of all concerned, we do not believe that mathematics education research as it has been carried and reported in the past in the Asia-Pacific region, or indeed around the world, has been particularly helpful or cost effective.[2] That is not the same thing as saying that contemporary mathematics education researchers have been striving merely to maintain the status quo. As Apple (1995) has pointed out, most educators see "mathematics as a field dominated by traditional pedagogies and conservative, social and pedagogical aims" (pp. 332–333), and there

1. In this book the term "Asia-Pacific region" will *not* encompass "Pacific Rim" nations on the American continents—such as Canada, Chile, and the United States of America. It *will* include islands in the Pacific Ocean as far west of Southeast Asia as Fiji. The "Indian sub-continent" will be taken to include Bangladesh, Bhutan, India, Nepal, Pakistan, and Sri Lanka. Note that the term is not well defined, and in fact UNESCO's Asia and Pacific region extends from Turkey in the West to Samoa in the East (UNESCO, 1989, 1992).

2. We recognise, of course, that there are no readily available criteria or benchmarks against which such a statement can be checked—see Sierpinska and Kilpatrick (1996).

are many contemporary mathematics education researchers who are carrying out research which has the potential to make the teaching and learning of mathematics more socially and culturally responsive. The reality is, though, as Apple (1995) has also stated, most people "experience mathematics as pure and uncontaminated by the real world (and taught in a strictly teacher-directed way)" (p. 333).[3]

In all parts of the world there are strong pressures on those engaged in mathematics education activities in formal and non-formal education settings to maintain the status quo. Rather than moving in a direction of increased autonomy, the daily lives of teachers in classrooms in many nations, including Asia-Pacific nations, are becoming "increasingly controlled, increasingly subject to administrative logics that seek to tighten the reins on the processes of teaching and curriculum" (Apple, 1995). "Quality classrooms, equity, collaboration, and empowerment of teachers and learners" might be the rhetoric, but the reality often expresses itself in increasingly centralised and standardised curricula and pedagogy, and in rationalised administrative procedures which rely heavily on evaluation procedures based on student achievement on pencil-and-paper tests (Apple, 1995; Ellerton & Clements, 1994; Secada, 1995). There is some evidence that this is adversely affecting the teaching and learning of mathematics (Stake et al., 1994).

Given that the Asia-Pacific region—as we have defined it—covers about half of the world's population, that mathematics is an integral part of the curriculum in all schools of the region, and that many billions of dollars are invested in the resources needed to support the teaching of mathematics each year by the nations,[4] it is clear that mathematics education research is (or at least should be) of fundamental importance. Given, too, that many believe that there is an intimate connection between the quality of a nation's scientific, technological and economic development and what goes on in the mathematics classrooms of its schools, the need for relevant mathematics education research of the highest quality can hardly be denied.

Should Mathematics Education Research be More Localised?

The arguments in favour of making mathematics education research more relevant and useful in the region, are made all the more compelling by the rapid developments towards making primary education compulsory for all school-age children in all the nations of the region. There are also ever-increasing proportions of children receiving a secondary education, and in some of the Asia-Pacific nations most students proceed to upper secondary school. Clearly, the need to implement the major tenets associated with the theme "Education for All" is being taken seriously by all nations of the region.

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3. Note that Apple (1995) went on to say that despite the history of conservatism among mathematics teachers and educators, the 1990s "is a time of considerable ferment in mathematics education, both pedagogically and socially" (p. 333).

4. Howson (1993) has estimated that, including overheads, expenditure on mathematics education would account for at least 10% of a nation's education budget (and expenditure on education might well account for 10% of a nation's budget).

There can be little doubt, however, that the major increase in participation in school education in the region over the past 20 years has been accompanied by the identification of complex mathematics curriculum issues. There are also several longstanding problems associated with the content and process of mathematics education research in some of the nations of the Asia-Pacific region. Who decides what research is needed, and which methods of research are most appropriate? And who is qualified to carry out the research?

In the introduction to their review of mathematics education research carried out in Malaysia between 1970 and 1990, Lai and Loo (1992) stated:

> A lot of national and international effort has been put into reexamining the mathematics curriculum and the strategies of teaching mathematics at both the elementary and secondary levels. As far as possible the formulation and planning of educational reform must be research based. Due to the shortage of local experts and researchers, it has been a common practice for developing countries to rely too heavily on imported ideas and knowledge. More often than not, the results turn out to be disappointing probably because the ideas and knowledge developed under different contexts are not suited to the needs of developing countries.To ensure better chances of success, the formulation and planning of educational reform should be based on local research studies. (p. 1)

The issues raised in this statement by Lai and Loo are important, and will be addressed throughout this book. We believe that new and different mathematics education research approaches are needed if the major issues associated with these changing patterns of schooling are to be adequately addressed (Ellerton & Clements, 1996; Pateman, 1996). It should not be assumed that mathematics education patterns and practices should be standardised in the different nations within the region.

Mathematics education researchers in the region cannot afford to be too "localist," however, for they need to keep abreast of global developments in their field. We are not convinced that enough mathematics education researchers in the region have participated in, or have even followed, contemporary debates on appropriate methodologies for conducting mathematics education research.[5] There are some, including Ramakrishnan Menon, of Singapore's Nanyang University, who have argued forcefully that statistical significance testing should not be used in mathematics education research (Menon, 1993). His paper stimulated the kind of debate on research methodologies that we hope this book will precipitate (see Bourke, 1993; Clements, 1993; Rowley, 1993).

One dilemma facing mathematics education researchers in the region is that although they should not be too "localist," significant cultural, linguistic, and education administration differences make it unwise for them to attempt to be too "generic," either. Bell and Kang (1994) emphasised this point in their comparison of factors— such as percentage of GNP spent on education, age at which children

5. Our basis for making this comment is the fact that of the nations in the region only three—Australia, Japan, and New Zealand—have been regularly represented at international mathematics education research conferences, such as the annual conferences of the International Group for the Psychology of Mathematics Education (PME).

commence school, average class size, hours of instruction in mathematics each year, mathematics curriculum differences, use of calculators in class, average number of children in families, and the employment of private tutors—which might be expected to influence mathematics learning in Australia and Korea.

Henriques (1996), in writing about education in the State of Madhya Pradesh in India, provided data which drew attention to the fact that the typical school in developed Western nations is very different from many schools in the Asia-Pacific region. Madhya Pradesh has a population of 66 million people and a literacy rate of 43 per cent—28 per cent for females and 58 per cent for males. Not only are there many villages without schools, but the children living in these villages have no access to any formal education program. Although 84 per cent of children attend primary school (98 per cent for males and 69 per cent for females), the drop-out rate in Grades 1–5 is reported to be 34 per cent, and "recent studies show that hardly 5 per cent of children at Grades 2–5 are attaining the minimum levels of learning" (Henriques, 1996, p. 354).

Henriques (1996) referred to a recent government survey which revealed that the number of classrooms in Madhya Pradesh was inadequate, buildings were dilapidated, and most schools had no teaching or learning aids except for poor quality blackboards. Very few books other than prescribed texts were in the school, sports equipment and musical instruments were not available, and basic necessities (like, for example, electricity, toilets, and playgrounds) were, more often than not, not present. Teaching kits provided by governments under various schemes were generally locked up, because teachers did not know how to use them and were afraid that they would be blamed if they were misplaced or damaged. Many of the teachers were poorly trained, had very heavy workloads, low self-esteem, and were looked down upon by society.

We could cite many other papers in which similar conditions in schools in various parts of the Asia-Pacific region are described (see, for example, Bhatanagar, 1996; Chauhan, 1996; Kamaluddin, 1996; Kizilbash, 1996). Our purpose for summarising some of the points made by Henriques (1996) about schooling in Madhya Pradesh, is not to belittle the often heroic efforts being made to educate children in that State; rather, we wanted to make it clear, at the outset, that education researchers who investigate mathematics education issues in the Asia-Pacific region are dealing with very different circumstances and must therefore expect to have different priorities from those researchers who collect data and analyse data from schools in relatively affluent Western nations. Of course, the cultures of groups living within economically poorer nations are often rich, and there is no guarantee that a form of school mathematics imported from the West will be acceptable to them or appropriate to their needs (Freire, 1971).

We do not want to create an impression, however, that all is well with mathematics education in so-called "developed" Western nations, where schools are comparatively well resourced and teachers have ready access not only to mathematics education researchers, but also to the results of mathematics education research. The truth is that in the West the findings of mathematics education research are often simply not known by teachers of mathematics. If they are known, they are frequently dismissed as being inappropriate or irrelevant to the teachers' situations.

In the early 1990s the veteran American education evaluator, Bob Stake, led a team of evaluators who looked at mathematics classes in a district in Chicago. They found that the teachers were working extraordinarily hard but nevertheless failing to match the expectations of the system. They were being told that research indicated that there were five key components of any good school mathematics program: (a) the education environment (especially characteristics of teachers and learners); (b) planning and programming; (c) teaching and learning; (d) monitoring and recording; and (e) assessing and reporting. The teachers already knew all this, but recognised that in many cases their mathematics lessons were dull, and that both they and their students were unable to cope with the demands being placed on them. Many students misbehaved or daydreamed; many teachers simply lived to survive the day. The burden of matching the expectations which society had of them, and keeping up with what was required of them by the education systems within which they worked, was proving to be too much (Stake et al., 1994).

Goudkamp (1996), an Australian primary school teacher, said the same thing in a recent article aptly titled: "Primary teaching ... Mission impossible." Goudkamp stated that she felt overwhelmed by the daily struggle of teaching a class of 29 Grade 3 children:

> I find it impossible to do the job as effectively or as well as I'd like given the size of the class, the limited access to resources, the lack of available assistance from support teachers, the impact of integration policies on my class, and the sheer number of hours needed to carry out the job ... The children in my class have a huge range of abilities and behaviours. I should be catering for each child's individual needs through my programming and teaching practices; I have to cater for students with attention deficit hyperactivity disorders, behaviour problems, those with learning difficulties, ... the talented and gifted; as well as the shy and withdrawn, the well motivated and the downright lazy. ... Assessment procedures should be in place so that I have a sound grasp of how well each child is doing in each key learning area (KLA) and in each strand within each KLA. I should be able to convey this knowledge to parents and care providers on a regular basis. (p. 15)

Warner (1996), an American secondary teacher, has written of similar pressures:

> The majority of my students, though in Advanced Placement classes, remained caught in a fairly rigid curriculum. With a standardized test at the end of the class measuring their performance as well as mine, and with their coming to me less prepared than their predecessors, my choices were clear: forget the idealism and eliminate the frills. ... Today I am met with curriculum demands so confining that few projects look possible. Nor am I sure that I can find the extra hours necessary to supervise, encourage, investigate and produce. But when the door to my classroom closes again and I am confronted by a new class of enthusiastic faces, I will probably try. To me, that is what it means to be a teacher. (p. 275)

What kind of mathematics education research is needed, then, not only to throw light on, but also to *improve* mathematics education practices around the world—and, in particular, in the Asia-Pacific region? Are there *any* generalisable theories, and/or research findings, which can be applied profitably across a range of diverse cultural and linguistically different settings in which the provision of resources varies greatly?

There can be no doubt that formidable challenges face mathematics education researchers working in the Asia-Pacific region. Perhaps the greatest challenge of all, though, is to develop strategies so that the research carried out will be *cost effective*. The term "cost effective" implies that the research will lead to genuine improvement in living conditions, enabling people not only to survive with dignity, but to have a greater degree of control over their own destinies. Given the circumstances, there are many who believe that there should be no place for investigations carried out by so-called "objective" researchers who do not work closely with teachers or with school communities. What is being called for is research which "builds on trust in teachers" and recognises that "behavioural and attitudinal changes on the part of the teacher can only be effectively sustained if the teachers are genuinely involved in the full planning and implementation" of projects (Henriques, 1996, p. 355).

What is Mathematics Education, and What is Mathematics Education Research?

Mathematics Education

Mathematics education is concerned with the development and implementation of appropriate mathematics curricula, and with all issues associated with the teaching and learning of mathematics. In keeping with the concept of lifelong learning, mathematics education covers learners of all ages and at all levels—from early childhood to adult. Thus, mathematics education is not solely concerned with curricula, classrooms, teachers and learners in schools; nevertheless, issues associated with school mathematics will be a major focus in this book.

Interestingly, although the boundaries between mathematics and mathematics education appear to be well defined, many so-called "mathematics education" programs contain little more than instruction in mathematical content. This is particularly common in courses for prospective or practising teachers in tertiary institutions in the Asia-Pacific region. Such courses often focus on the revision of senior secondary mathematics topics or incorporate first-year university mathematics topics like Elementary Set Theory and Vectors. In South Korea, for example, it has been reported that only 38% of the departments of mathematics education in universities have different curricula from the departments of mathematics in the same universities (Young-Han Choe, 1995). Furthermore, there is little communication between those teaching in different mathematics education departments in South Korea.[6]

6. Note that such views are not confined to so-called "developing," or "recently-developed" nations. We are aware of similar views being held in some universities in the United States of America, the United Kingdom, and Australia.

Mathematics Education Research

Mathematics education research involves designing, conducting, interpreting and reporting research in the domain of mathematics education. Mathematics education research is not confined to questions such as "How should one teach fractions to Grade 6 children?" or "Is a unit circle approach to teaching Trigonometry better than the traditional right-angled triangle approach?" Although such content questions can form the basis of legitimate research investigations, there are now large international mathematics education research literatures pertaining to many other non-content areas. These include, for example, gender and class equity, problem solving, constructivism, co-operative group learning, language and cultural influences on learning, implications of hi-tech developments for contemporary mathematics curricula, programming for children who are mathematically precocious, and the development of more appropriate assessment procedures for school mathematics.

The rapid economic and technological development of most of the nations in the region, and the large numbers of contrasting languages and cultures within and between the nations of the region, point towards the necessity for developing and implementing different mathematics education practices in different contexts. Any approach to mathematics education which assumes that school mathematics should be predominantly concerned with preparing students for the kind of higher mathematical studies taken by tertiary students in "advanced" nations, such as the United States and Japan, is likely to be unacceptable, even within the "advanced" nations. Yet, it could be argued that many Asia-Pacific nations have consciously (and often unconsciously) imported and adopted mathematics curricula, and teaching, learning and assessment practices which are not appropriate (Ridgway & Passey, 1993).

It could also be argued that over the past 20 years Western educators have not only got their own mathematics education equations wrong, but have passed on their errors to education systems around the world. Altbach, Arnove, and Kelly (1992), Carnoy (1974) and Zachariah and Silva (1980) have all argued that national school systems need to be seen as located within unequal power relations among nations. According to Kitchen (1995), through design, historical circumstances, and the unequal distribution of material and intellectual resources, Western industrialised countries have dominated, and continue to dominate, the economic and educational systems of less industrialised nations. Coombs (1985) has argued that the historical roots of this unequal relationship date back to colonial times when many Third World countries adopted transplanted educational models from industrialised countries which "grotesquely misfit the countries' actual needs, circumstances, and resources" (p. 33).

More specifically, Ridgway and Passey (1993) have asserted that the continued and widespread use of externally-set, written examinations in mathematics is a vestige of colonial power, and even if such examinations are set within-country, act as agents of cultural imperialism. According to Ridgway and Passey, external examinations offer affluent members of society access to universities worldwide, and for the less well off, access to skilled jobs. Much more will be said, in Chapter 6 of this book, about the findings of research into the history and effects of external ritten examinations, across the world.

In relation to the purposes of this present book, it should be noted that every nation in the Asia-Pacific region except one—Thailand is the exception—has, been, politically speaking,[7] a "colony" at some stage over the past 100 years (Bray & Lee, 1996). From that perspective, colonialist tendencies have been, and in fact continue to be, of particular importance in all aspects of society, including mathematics education and mathematics education research.

Culture, Mathematics, and Mathematics Education

In the context of mathematics education, avoiding the external imposition or internal acceptance of colonialist practices is easier said than done. Secada (1995) might extol the virtue of people defining their own issues in their own ways, from their own perspectives, using their own terms, but in the twenty-first century, no nation will be isolated from all other nations, no community from all other communities. There will always be influence from "outside," and it is unlikely that people left to make their own decisions about the forms of mathematics education they want for their schools, and for their children, will choose anything other than what is being taught in "First World" countries. That form of mathematics is sometimes thought to be the secret ingredient by which Western nations have "progressed."

The often misplaced faith which parents have in the willingness and capacity of their offspring to cope with the abstract demands of formal mathematics makes it difficult to develop research-based policies in relation to mathematics education. On the other hand, teachers and administrators in education institutions develop and apply policies which determine who is allowed to study what form of mathematics to what level. In local situations, emotions, values, and politics, are usually more influential than reasoned judgement in regard to decisions on forms of mathematics education which are made available to learners. It is for this reason that mathematics education research needs to embrace a range of methodologies. A cognitive approach, or indeed any other single approach, by itself will explain only part of an overall picture. What is needed is some kind of anthropological overview of the various *cultures* which permeate the environments in which mathematics learning occurs, and, in particular, the cultures of schools and other formal education institutions, in which mathematics education programs are to be found. Of course, that is an impossibly large task, even though this book will pay special attention to education in the Asia-Pacific region.

Noss and Hoyles' (1996) Vision of "the Mathematical"

Noss and Hoyles (1996) have commented that in their attempt to steer a course between furthering their own understanding of mathematical learning, "and

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7. Of course, if a broader definition of "colonialism" is permitted, then colonialist practices might be regarded as occurring in nations which have never been politically colonised. (See Clements, Grimison, & Ellerton (1989) for a discussion of this broader approach to the concept of colonialism in education.)

revisioning what mathematical learning comprises," they have reached the conclusion that "the key to this complexity is to take seriously the notion of culture" (p. 7). They stated that they could not "analyse all the inter-relationships between learners, teachers, and mathematical knowledge," but at best they could try to understand the complexity of lived-in-cultures from what they were able to see. They have chosen to present a socio-cultural perspective in which the role of technology, and especially the role of the computer, is problematised.

According to Noss and Hoyles (1996), the view of mathematics education and mathematics education research emanating from Grouws's (1992) *Handbook of Research on Mathematics Teaching and Learning* is American. Their own book—in which they identify and elaborate upon interacting forces influencing mathematics curriculum change and mathematics education cultures—provides a provocative and refreshing approach to reconceptualising the major elements of the mathematics education and mathematics education research fields.

In reviewing relationships between qualitative and quantitative research methodologies, Noss and Hoyles' (1996) referred extensively to how they were to able to exploit the computer's presence in education settings. Our focus, however, will be different insofar as we will attempt to set current debates on the role of mathematics education research in historical, philosophical, comparative, social and mathematical contexts.[8] Furthermore, the book attempts to provide a survey of mathematics education research in the Asia-Pacific region—which, as far as we know, has not been undertaken before.

The Internationalised Culture of School Mathematics[9]

The term "culture" generally refers to social heritage—those characteristic behaviours which are transmitted from one generation to the next. Although the notion of culture includes collective artifacts such as symbols, ideas, beliefs and aesthetic perceptions (and mathematics has recognisable forms of each of these), it also embodies the distinctive forms of discourse, behavioural norms, and common patterns of the organisation of institutions, groupings, and rituals. As Popkewitz (1988) has acknowledged:

> Schools function according to rules and procedures that provide coherence and meaning to everyday activities and interaction. Such rules and procedures are embodied in the regularised patterns of behaviour, specific vocabularies and particular roles that we associate with schooling. (p. 222)

Although we generally think of national and racial identity as, to a large extent, defining culture, institutions such as the institution of schooling, have well-established and powerful cultures which are maintained and transmitted across generations. What these institutions share in common is a general self-awareness and a sense of boundaries. Particular patterns of socialisation, and

8. This approach is consistent with the seventh of our ten propositions for reconceptualising mathematics education research—see Chapter 7 of this book.

9. The text is this section is based on the discussion of "culture in education" in Mousley and Clements (1990), and in Hamilton (1976).

systems of "reinforcement" of the prescribed values and behaviours are recognised as giving an organisation a life of its own (Wallerstein, 1990). In nations with well-developed patterns of schooling, the values and behaviours which influence classroom practice are interconnected and are often interdependent. Mathematics education, in particular, has sufficiently well-entrenched norms that it can be regarded as a constituting a "culture" in its own right.

The Typical Western Mathematics Classroom

In the United States of America, Good, Grouws and Ebemeier (1983) reported that certain kinds of mathematics teaching behaviour was likely to result in higher student achievement on standardised pencil-and-paper tests. According to Good et al. (1983), a typical 40-minute mathematics lesson would be likely to lead to "greater" student achievement if the lesson were conducted in a whole-class (as distinct from small-group) manner and proceeded according to the following structure:

1. Lesson starts with the teacher marking set homework and reviewing past work. Brief mental computation exercises are also given (8 minutes);
2. Lesson develops, with the teacher establishing comprehension of skills and concepts (20 minutes);
3. Students do seat work—individual (15 minutes);
4. Lesson concludes with a brief overview, and homework is set (2 minutes).

Commentators (for example, Sullivan, 1989) have not only questioned the fixed nature of this lesson plan, but have also questioned whether achievement on pencil-and-paper mathematics tests constitutes an acceptable criterion for measuring lesson effectiveness. Nevertheless, it is likely that mathematics lessons in the schools of many Western nations follow similar patterns to that suggested by Good et al. (1983) (see, for example, Bourke, 1984; Clarke, 1984; Ernest, 1989; Mousley & Clements, 1990). By contrast, it appears to be the case that in many Asian nations, mathematics teachers adopt different formats for their lessons (Stigler & Baranes, 1988).

The important point to note here is that there is no compelling logic behind the format identified by Good et al. (1983), or behind that of any other pattern adopted by a particular teacher of mathematics. The reality of school mathematics is that it involves not only the acceptance of teacher and school decisions on content, but also acceptance by (a) learners of the patterns of teaching which are developed by the teachers; and (b) teachers of the patterns of classroom behaviour in which their students engage. Mathematics classrooms become mini-worlds which exhibit fairly well-defined patterns of behaviour, and of pre-determined standards of reasoning. Whether we like it or not, in each classroom teacher-student and student-student interactions take place in reasonably well-defined ways.

Furthermore, as Popkewitz (1988) has stated, "the social patterns of school conduct are not neutral but related to the larger social and cultural differentiation that exists in our societies" (p. 221). The socialisation effects associated with

expectations arising out of the curriculum, assessment traditions, and teacher-student-subject interconnections are so powerful that "despite having different beliefs about mathematics and its teaching, teachers in the same school are often observed to adopt similar classroom practices (Ernest, 1989, p. 253). Not many teachers of mathematics feel free to reject the mathematics education culture which surrounds them.

Critiquing the Traditions of Mathematics Education

Why have we devoted so much attention, in this first chapter of a book primarily concerned with mathematics education research, to a discussion of the concept of culture, and of how this relates to mathematics and mathematics education? The reason is simple: it is cultural factors which have the most pervasive influence on assumptions and practices in the realm of mathematics education. As Garaway (1994) has argued, education researchers seeking to explain, for instance, failure to achieve, are focusing less on the characteristics of learners and teachers and more on education institutions and the conventions under which they operate. Academia is increasingly being viewed as a system of interrelated cultures, and instead of focusing on deficit or difference interpretations for failure, researchers are attempting to identify underlying assumptions and approaches within an education institution or society which generate failure and failure mentalities.

According to Garaway (1994), the aim of mathematics education researchers should be to improve mathematics education, in general, and school mathematics in particular. In order to do that they will need to make explicit the patterned elements of particular mathematics education settings—the norms, standards, expectations, and beliefs which influence the behaviour of the teachers, students, parents, and others who are involved (Hamilton, 1976). They will need to identify the hidden curricula, the "silent languages" of mathematics classrooms, and the tacit rules and unspoken transactions which serve to regulate mathematics classrooms.

More and more researchers are adopting the procedures of the anthropologist in an attempt to gain a holistic picture of the factors which influence mathematics learning (see, for example, the chapters in the collection edited by Secada, Fennema, & Adajian, 1995). If, like critical theorists, they wish to do more than that, and seek to improve mathematics education, then they will need to do more than what anthropologists do—they will need to set about finding ways of modifying cultures, of changing entrenched behaviours, and of influencing ways of thinking. Chapter 5 of this book, which is concerned with action research, presents an overview of one type of response by educators seeking to modify cultures which they deem have unjust elements.

In this book we shall not proceed from the viewpoint that the values inherent in any particular culture of mathematics education should not be questioned by anyone outside the culture. Thus, for example, in Chapter 6 we point out that in many Asian cultures pencil-and-paper tests are vitally important components of tures for which status and family pride are very much dependent on

examination success. We then proceed to argue, from research data, that the faith within Asian societies in the objectivity and reliability of the results of pencil-and-paper mathematics tests is misplaced.

This approach is in line with Bruner's (1986) comment:

> Culture is constantly in the process of being recreated as it is interpreted and renegotiated by its members. In this view of culture, a culture is as much a *forum* for negotiating and renegotiating meaning and for explicating action as it is a set of rules ... It follows from this view of culture as a form that induction into the culture through education, if it is to prepare the young for life as it is lived, should also partake of the spirit of a forum, of negotiation, of the recreation of meaning. But this tradition runs counter to traditions of pedagogy that derive from another time, another interpretation of culture, another conception of authority—one that looked at the process of education as *transmission* of knowledge and values *by* those who know more *to* those who know less and know it less expertly. (p. 123)

As Freire (1971) has stated, the world is not static, and members of society, and especially teachers, can discover that they are creators, rather than mere perpetrators, of culture.

However, as Gregg (1995) has pointed out, the robustness of traditional practices in school mathematics raises the fundamental question of why those traditional practices have been so constant and durable. Gregg (1995) analysed how a beginning high school teacher was acculturated into a school mathematics tradition (that is to say, the beliefs and practices which characterise traditional approaches to the mathematics program in that school). His analysis suggested that teachers assist learners to cope with the tensions and contradictions which underlie the beliefs and practices of school mathematics. In addition, the explanations they give for why they choose to cope in the ways they do, actually help to sustain and reinforce the tradition.

The latter-day Jerome Bruner (1996) has felt moved to comment on the importance of communication, and on how it has been, until recently, neglected by education researchers:

> There is one "presenting problem" that is always with us in dealing with teaching and learning, one that is so pervasive, so constant, so much part of the fabric we often fail to notice it, fail even to discover it—much as in the proverb "the fish will be the last to discover water." It is the issue of how human beings achieve a meeting of minds, expressed by teachers usually as "how do I reach the children?" or by children as "what's she trying to get at?" This is the classic problem of Other Minds, as it was originally called in philosophy, and its relevance to education has mostly been overlooked until very recently. In the last decade it has become a topic of passionate interest and intense research among psychologists, particularly those interested in development. (p. 45)

This matter of communication has been taken up by mathematics education researchers, especially (but not only) by those investigating classroom cultures.[10]

10. The nature of communication is an important common theme in the writings of radical constructivists, who emphasise the weaknesses of old transmission theories of learning (Steffe, 1990; von Glasersfeld, 1991).

Can Mathematics Education Research be Justified?

If the cultures of school mathematics in nations in the Asia-Pacific are to be examined, then which criteria should we employ in order to make decisions on whether a culture, or an aspect of culture, needs to be improved. In Chapter 5 we shall argue, as have Kemmis and McTaggart (1988), that the main criterion should be to improve the "rationality and justice" of practices in mathematics education. Notice that this criterion makes no *explicit* mention of economic gain. That might be deemed to be naive, given the economic rationalist approaches to education which politicians, economists, and education bureaucrats have popularised in many nations since the late 1980s (Ellerton & Clements, 1994).

Many of the economies of the nations in the Asia-Pacific region are fragile, to say the least, and the question of whether government spending on mathematics education research is justifiable must be faced. Can any government really justify spending large amounts of money on such research at a time when there are not enough funds to ensure that everyone can afford food, clothing, housing, basic education, and welfare? Large sums of money are needed to build, maintain, and staff hospitals, schools, railways, and other public facilities, roads need to be built, and environmental issues attended to, ...

We believe that this kind of thinking is shortsighted, and any "savings" derived from policies which deny substantial funding for mathematics education research would represent false economy. If governments accept the idea that every child has a right to at least a primary education, then money spent on providing the resources needed to meet this commitment will be wasted if the curricula and the teaching and learning processes do not help children to become better and more productive citizens. If huge amounts of public funds are to be spent on mathematics education, if all children are to be expected to learn mathematics every day for many years, and if the development of a mathematically literate population is linked with economic growth and with improved standards of living, then research aimed at improving the quality of the teaching and learning of mathematics would appear to be absolutely essential.

In this context it is significant that the Asia-Pacific region, as we have defined it, accounts for over one-half of the world's illiterates, with Bangladesh, China, India, Indonesia and Pakistan having especially large numbers According to Promkasetrin (1991–1992), one of the main obstacles to faster progress in improving world literacy is "economic conditions in the majority of countries which have low literacy rates" (p. 20).

There is much evidence that school mathematics as we now know it in virtually all countries around the world is simply not working for many children. It has been tried in the balance and found wanting. According to Promkasetrin (1991–1992), a significant proportion of primary school-age children in the Asia-Pacific region either do not attend school at all or, if they do, they leave after a few years without having acquired basic literacy and numeracy skills. Many adults are illiterate and functionally innumerate, but non-formal education programs in mathematics are often too abstract to help much. In many of the nations of the Asia-Pacific region

girls are under-represented in enrolments at every level of education, and in mathematics education a very much higher proportion of male students than female students study non-terminal senior secondary or tertiary mathematics subjects. This is related to the fact that the majority of out-of-school youth in developing nations in the region is female (Promkasetrin, 1991–1992).

From this perspective, this book on mathematics education research, does not deal with esoteric, academic matters. Indeed, it could be argued that the need to reform a situation whereby many children in the Asia-Pacific region are not given adequate opportunity to learn, and therefore to apply, mathematical skills represents one of the world's most pressing problems. Just as "culture" is one of the major themes of this book, so too is "equity."

A UNESCO (1989) report described equity in education in the following terms:

> **Equity.** Equity of access, equity of treatment, and equity of results. The main thrust in equity of access is to reach disadvantaged groups—e.g. girls, those living in remote rural areas, and those living in urban slums. Equity of treatment concerns studies which reveal that many teachers often do not treat students equally in the classroom. It also is noted that the curriculum is not always sufficiently sensitive to each other's diverse cultures. Over time the values of the middle class have become the norm of the education system. The problems that stand in the way of achieving equity in academic results are mainly due to inequities of access and treatment. (UNESCO, 1989, p. 2)

As this passage indicates, it is impossible to divorce equity from historical and cultural considerations. Root causes of inequality need to be identified and, somehow, eliminated.

Problems arising from inequity are not confined to developing nations, of course. In schools in working-class or minority group areas in developed Western nations, many teachers of mathematics and their students are struggling to see much relevance in the prescribed rather abstract mathematics curricula which they are required to follow (Stake et al., 1994). Fundamental changes are required: there needs to be a reconceptualisation of the purposes of mathematics education at all levels (the "why"), what mathematics should be studied (the "what"), how it should be presented and how it should be assessed (the "how") (Ellerton & Clements, 1989a).

We are concerned that a continuation of much of what has gone on under the general umbrella of "mathematics education research" in Asia-Pacific nations during the last quarter of the twentieth century has been narrow, almost sterile, and unlikely to throw much light on difficult and important mathematics education issues. More balanced mathematics education research programs, incorporating a much wider range of methodologies, need to be developed within the different nations.

Although there are many important education research agencies within the Asia-Pacific region, most of these agencies do not have staff who are recognised members of the international mathematics education research community. A consequence of this fact is that much of the "official" thinking about mathematics education research in the region has been done by educators and bureaucrats who

have little background in mathematics education research, and who therefore have had only a scant knowledge of the literature and key research findings in the field.[11]

Towards More Balanced Mathematics Education Research Programs

Critiquing Education Research Methodologies

Two decades ago, much of the research carried out in the domain of education, including mathematics education, was quantitative in nature, involving the use of inferential statistical procedures to generalise from samples to populations. Implicit in such research was the notion that carefully designed studies could reveal certain generalisable truths pertaining to education. Much of this kind of thinking is still present in the Asia-Pacific region today (see UNESCO, 1992).

In the 1990s, however, there are many mathematics educators who have little faith in the results of education research studies which have employed statistical significance testing. Some writers have strenuously advocated that inferential statistics should never be used in education research (Carver, 1978; Haig, 1996; Menon, 1993; Schmidt, 1996). Simultaneous with this decline in the emphasis on and acceptance of the use of inferential statistics in education research, there have been strong moves towards exploratory data analysis techniques incorporating graphical approaches (Biehler, 1989; Jones, 1993; Tukey, 1977). One of the reasons for the move away from the use of inferential statistics has been the influence of so-called "postpositivist" theorists who have used the pejorative terms "positivist" and "reductionist" as labels for quantitative research in which complex statistical analyses have been employed. Postpositivist writers have claimed that "any adequate approach to education enquiry must reject positivist notions of rationality, objectivity and truth" (McTaggart, 1991, pp. 33–34).

Traditional anthropological and ethnographic methods of education research have also been the subject of criticism. Postmodern theorists, for example, have argued that most of the qualitative, interpretive research carried out by education ethnographers has accepted (and unwittingly confirmed) the status quo in order to interpret it. In this way, critics claim that such research has failed to identify those aspects of the existing social order which frustrate the pursuit of rational and equitable goals (Carr & Kemmis, 1986).

Opponents of ethnographic research have been especially critical of researchers who have tended to adopt interpretive categories which are not in harmony with values in the cultures being investigated. Thus, for example, developmental research in which children from many countries were asked to respond to classical

11. A UNESCO (1992) publication included names of 73 agencies in 16 Asia-Pacific nations (Australia, Bangladesh, China, India, Indonesia, Iran, Lao PDR, Malaysia, Myanmar, Nepal, Papua New Guinea, Philippines, Korea, Sri Lanka, Thailand, and Vietnam). Some important education research agencies/centres in Australia, Brunei Darussalam, Japan, New Zealand, and Singapore, for example, were not listed.

Piagetian conservation tasks, and were then classified as being at particular Piagetian stages, has been criticised as being colonialist and potentially racist (Zevenbergen, 1993).

Critiques of Piagetian research and methodologies represent part of a wider move in education away from theory-*driven* research towards theory-*generating* research (Strauss & Corbin, 1990). The general thrust of these critiques is that it is now recognised that there are so many complex variables associated with education that it is naively simplistic to assume that cognitive development and learning can be readily explained by a single overarching theory developed by studies based on samples which can never be truly representative.

In the 1980s and 1990s, advocates of action research in education (see, for example, McTaggart, 1991) have maintained that the *only* legitimate and economically justifiable form of education research is *action research*. For, it is claimed, it is *only* action research which aims directly to *improve* education structures and practices through the democratic collaboration of practitioners.

Almost every contemporary commentator on education research states that there is no firm dividing line between quantitative and qualitative research, that the two approaches to research should be regarded as complementary, and that in any case the boundaries between the two are often blurred. A UNESCO (1992) report stated that quantitative research tends to be "obtrusive, controlled, objective, and product-oriented," whereas "qualitative researchers favour methods which are naturalistic, uncontrolled, subjective and process-oriented" (p. 136). However, with respect to quantitative research it needs to be recognised that the 1990s has witnessed a major questioning of the legitimacy of Fisherian statistical significance testing (for example, with t-tests and analysis of variance), and that many of the pre-1990s treatises on quantitative education research methodologies are, to say the least, out of date—and possibly even misleading. More will be said on this later in this book.

We believe, though, that those who advocate that any *particular* form of education research is best for *all* circumstances are misguided. To illustrate our point, we wish to summarise and comment briefly upon comparative research carried out over the last two decades into students' mathematical achievement in China, Japan, and the United States of America.

Quantitative Studies can be Important

Large data sets pointing to the conclusion that Chinese, Japanese and Korean primary school children tended to outperform North American primary school children in mathematics have been reported (see, for example, Chuansheng, Lee, & Stevenson, 1993; Song & Ginsburg, 1987; Stevenson, Lee & Stigler, 1986). The educational significance of such research has, however, been questioned on the grounds that too much emphasis was placed on mere computational achievement.

In a study which set out to investigate whether superior Asian achievement was mainly in the area of computation, Stigler and Baranes (1988) tested first- and fifth-grade children in Sendai (Japan), Taipei (Taiwan), and Chicago, using ten tests covering computational skills, mental calculations, memory for numbers, novel mathematical word problems, conceptual knowledge (across a wide variety of nains including place value, equations, and fractions), operations, estimation, phing, measurement, visualisation, and mental image transformation. Data

were obtained from interviews as well as from pencil-and-paper tests, and the instruments were judged to be culturally unbiased by a team of researchers representing each of the cultures being studied.

Results from the exploratory data analyses were clear and dramatic: Japanese students outperformed American students on almost every test, and Taiwanese children were usually not far behind the Japanese children. Similar results were found in other comparisons of mathematical problem-solving performance between North American and Asian school children (see, for example, Song and Ginsburg's (1987) data comparing Korean and U.S. school children).

The point to be noticed here is that Stigler and Baranes's (1988) comparisons of performance on the various tests essentially employed contemporary exploratory data analysis procedures. That is to say, although the analyses were essentially quantitative, they were reported in terms which were easily understood.[12] However, the report of this study has been very influential both within and outside the United States of America. The message is clear: quantitative education research can be powerful and influential if its results are communicated in suitable language.

Qualitative Classroom Investigations can be Important

Another interesting feature about the Stigler and Baranes (1988) study is that the study involved both quantitative and qualitative components, each complementing each other. The main research question addressed in the qualitative study was: Why do Japanese children and children from certain other Asian countries outperform American children on mathematics and mathematics-related tests? This question called for an examination of the impact of cultural differences with respect to the way mathematics is taught and learned in different cultural settings. In particular, a comparative qualitative study was needed into similarities and differences in discourse patterns evident in mathematics classrooms in the different countries.

Tsang (1988), who studied the mathematics achievements of immigrant students from China, Vietnam and Hong Kong who had been in the United States of America for less than two years, also used a combination of research methods. In addition to quantitative and qualitative methods, Tsang undertook eight case studies of individual students. Data from these case studies revealed that all eight students were from countries with very competitive educational systems, and in order to gain entrance to higher levels of schooling they had been required to pass a series of public examinations.

Another of Tsang's (1988) findings was that in their Asian schools the eight case study students had been expected to solve many problems quickly and accurately, whereas in the United States, teachers tended to give more verbose explanations. Also, in the United States the students appreciated being able to work together cooperatively in class—they often used problem-solving sessions to share with each other rules which they knew would solve the problems even though their teachers had not articulated them succinctly.

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12. Stigler and Baranes used exploratory data analysis procedures rather than complex, multivariate analyses. This made their paper much more accessible to many readers.

Like Tsang (1988), Stigler and Baranes (1988) found interesting differences between the cultures of the classrooms in the different nations. In Grade 1 mathematics classrooms in Japan, almost 50 percent of all instructional segments contained verbal explanations by either the teacher or a student. By contrast, this was the case in only 20 percent of the North American Grade 1 mathematics classrooms involved in the study. Stigler and Baranes (1988) commented:

> This emphasis in Japan on verbal discussion of, and reflection upon, mathematical topics—so rarely found in American elementary school classrooms—is also evidenced in other ways in our observations. For example, the most common means of publicly evaluating student work in the Japanese classrooms was to ask a student who obtained the incorrect answer to a problem to put his work on the board, and then to discuss, with the entire class, the process that led to the error. The most common form of evaluation in the American classrooms, by contrast, was simply to praise a student who answered the problem correctly. By focusing on errors, Japanese teachers have a natural basis on which to build a discussion: praise, on the other hand, functions as a conversation stopper. (p. 296)

Chuansheng et al. (1993) reached similar conclusions with respect to qualitative between-country differences for mathematics classrooms. According to Chuansheng et al. (1993):

> Rather than beginning a lesson with definitions and descriptions of operations, Chinese and Japanese teachers frequently begin a lesson by describing a practical problem. The remainder of the class period is spent in finding various approaches to its solution. Only at the end of the period are the formal properties of the new procedures defined and described. In a word, Asian teachers strive to present mathematical concepts in a meaningful context. (p. 33)

Data from the qualitative studies helped to dispel myths about the teaching of mathematics in Asian schools. Once again, the words of Chuansheng et al. (1993) are apt:

> The stereotype, both in the East and the West, is that Asian students have fared well in international comparisons because they are well-drilled in the basic concepts and operations of mathematics, but that they lack the ability to apply these effectively in problems that do not rely on the routines in which they have been rehearsed. This was not the case in our studies.
>
> Asian teachers do not rely on drill and rote learning as techniques for teaching mathematics. This may have characterised teaching practices many years ago, but it is not a valid description of what occurs today. At the same time, contemporary teaching practices do not embody esoteric or exotic procedures unknown in the West. What we have found is that Asian teachers apply what everyone would agree are reasonable and thoughtful approaches to presenting mathematics to young children. The major difference between what occurs in the East and the West is that Asian teachers appear to apply these approaches with greater consistency and frequency than do their Western counterparts. In doing this, they are helping to produce students who are leading the world in their achievement in mathematics. (p. 33)

Stigler and Baranes (1988) reached similar conclusions. They reported that the ɔhases on verbal explanation in Japanese schools did not mean that Japanese

teachers relied less on concrete manipulatives than did their American counterparts. Indeed, Japanese and Chinese teachers were reported as using more concrete aids and providing more real-world problem-solving scenarios in their classrooms than their American counterparts. Also, whereas in Japanese classrooms the frequency of verbal explanations rose markedly when manipulatives were used, this was not the case in American classrooms. Japanese teachers used concrete aids to stimulate discussion about mathematical concepts, but American teachers tended to allow such aids to become a substitute for discussion (Stigler & Baranes, 1988).

Stigler and Baranes (1988) also reported that Japanese mathematics lessons appeared to move at a more relaxed pace than American lessons. Whereas teachers in Japan were often observed to spend an entire 40-minute lesson on one or two problems, in the United States, only 17 percent of lesson segments lasting 5 minutes were concerned with one problem. (The corresponding percentage in Japan was 75.) From Stigler and Baranes' study it might be concluded that American teachers deny students opportunities to discuss mathematical concepts and problems—note however, that Tsang's (1988) case study data suggested that there was more chance for co-operative learning in mathematics classes in North American schools than in schools in China, Hong Kong, and Vietnam.

The patterns which Stigler and Baranes (1988) identified in North American mathematics classrooms can certainly be found in other countries (e.g. in Scotland—see Ruthven, 1980; and in Australia—see Clarke, 1984). Bishop (1988) has referred to the need for teachers of mathematics to provide opportunities for reflection in order "to allow for connections to be made between the ideas from one activity and those from another" (pp. 136–137). Apparently, this does not happen in many mathematics classrooms in Western nations.

The finding that there are fundamental qualitative differences between what transpires in Asian and American mathematics classrooms provides cause for reflection by mathematics educators and teachers around the world. In particular, differences in the nature and quality of whole-class instruction in different countries would appear to be educationally significant. The most important point to be noted here, however, is that the quantitative and qualitative analytic procedures employed in the Stigler and Baranes (1988) and Tsang (1988) studies complemented each other, and that the findings of these studies have provided the basis for future investigations into significant education issues.

It should also be noted that in Stigler and Baranes's (1988) comparative studies into characteristics of Asian and North American mathematics classrooms, a team of researchers was involved in systematic and careful data collection. Trained observers provided written descriptions of what was going on in the mathematics classes, with both verbal as well as non-verbal behaviour of teachers and students being recorded. Information written on the blackboard as well as descriptions of concrete materials used by teachers and students during the lesson were also noted (Chuansheng et al., 1993).

Cultural Contexts of Education Need to be Studied

Stigler and Baranes (1988) also included in their research design a comparative investigation into the attitudes and beliefs of students, teachers and parents. They

found that there were enormous cross-cultural differences in beliefs about ability and disability. Asian mothers, for example, when confronted with data indicating that their children were having difficulties at school, tended to give their children more help with their homework, whereas American mothers tended to express hopelessness about prospects of helping their children, and merely tried to encourage them.

Almost every writer on Asian education has stressed the importance of family influences on education achievement. White (1987) has stated that if Westerners need to "borrow anything from the Japanese it is, paradoxically, the attention they devote to their own, paramount, cultural priority: the improvement of children's lives" (p. 191). Chuansheng et al. (1993) discussed the same issue with respect to Chinese and Japanese families:

> Answers to why Chinese and Japanese students do so well in mathematics involve complex interactions between culture, family life, and teaching practices. Cross-cultural studies of academic achievement help to illuminate these interactions and offer interesting examples of how teachers, parents and students can combine their efforts to advance students' progress in mathematics. It is difficult to transfer some of the practices that are highly dependent upon culture, but others, such as teaching practices, may offer useful models for what could be adopted in the West. (p. 34)

Formal education systems cannot be divorced from the cultures in which they exist. Inevitably, children's learning styles, beliefs, self concepts, preferences, and cognitive schema are linked with the cultural milieu which surrounds them. That is why many education researchers are coming to believe that education research which is based on overarching grand theories of development and learning are ill-founded and potentially dangerous (Ellerton & Clements, 1996a). This is the very reason why anthropological research which seeks to place education systems within their cultural contexts is needed.

An interesting case which can serve to illustrate the importance of the need for mathematics education researchers to give due accord to cultural factors in their research has arisen from a 1993 decision of the Malaysian Government that all Malaysian primary school pupils should be taught to use the abacus. This decision was prompted by the strong performance on externally-set mathematics examinations by children from Chinese-background families living in Malaysia. Research by Liau Tet Loke and Hong (1994) suggested that a majority of Malaysian teachers—from Malay, Chinese and Tamil backgrounds—were inclined to believe that using an abacus could help primary school children to develop better skills in numeracy.

However, the superior performance of Chinese students on pencil-and-paper tests could be the result of many different factors—such as the cultural priority accorded to the preparation of children for competitive written examinations, and the different relative value traditionally placed on the study of mathematics by Malay, Chinese, and Tamil families. Although the relative influence of different variables will need to be sorted out, one could safely predict that it will be cultural variables which will be shown to be of paramount importance.

Participatory Research Involving Teachers is Important

In summarising their research comparing mathematics instruction in Asia and in the United States of America, Stigler and Baranes (1988) commented that their findings should cause educators to question several assumptions that underlie American mathematics education:

> First is the assumption that direct explanation is not a useful way to teach mathematics to young children. There is a widespread belief in our society that concrete experiences are the best way to teach young children, and that language will either go over their young heads, or lead to learning of a superficial kind. There are two important findings from the Japanese observations that we need to ponder: (a) Young children are capable of responding to, and apparently understanding, complex verbal explanations; and (b) it is possible to stress both concrete experiences and verbal explanations at the same time. It is possible, in fact, that both are necessary to promote high levels of learning. (p. 299)

The statement clearly implies that teachers in the United States of America should at least try to emulate some of the teaching and learning approaches which were identified as successful in Asian schools. The question arises, though, whether it makes sense to attempt to translate these approaches to very different education contexts. If the answer to this question is "yes," then the further question arises which teachers should be involved in the trialing of the imported approaches, and the conditions under which the trials should be conducted.

Increasing numbers of Western educators believe that action research provides the only model whereby significant change in agreed directions can be achieved. Chapter 5 of this book will focus on the application of action research in education settings. Here it suffices to say that although many different definitions have been given for the term "action research," the characteristic features which have made the largest positive contributions to achieving change are: (a) all members of an action research group participate on a voluntary basis; (b) the group acts in a collaborative and democratic manner at all times, with no member being more important than any other; (c) meetings are held regularly, and minutes kept; and (d) the aim is to improve education, where the criteria for improvement rest in the group's interpretations of what needs to be done to achieve greater equity and justice.

Thus, if, for example, some teachers at a school in Australia (or Fiji, or Thailand, or ...) were impressed with what they heard and read about Japanese mathematics classrooms, they might decide to work together as an action research team in an attempt to employ the methods used by teachers in Japan. There is a growing body of research which suggests that although such a decision would maximise their chances of achieving the change they desired, nevertheless their task would be still be a difficult one (Robinson, 1989).

Philosophical and Historical Research can be Important

It is easy to overlook the importance and role of mathematics education research which identifies and analyses philosophical and historical foundations which affect the teaching and learning of mathematics. Without insights deriving

from research of this kind, it is difficult to make wise decisions about the directions any major research thrust should take.

There can be little doubt that although the constructivist movement in mathematics education (see Cobb, 1994a; von Glasersfeld, 1991a, 1995) continues to have a major influence on contemporary mathematics education, there are some mathematics educators and education philosophers who believe that radical constructivist philosophies in mathematics education lack coherence or completeness (Ellerton & Clements, 1992a; Kilpatrick, 1987;Lerman, 1996; Matthews, 1995; Suchting, 1992).

Barton, Begg, Butel and Ellis (1995), for example, maintain that New Zealand's new curriculum is based on a constructivist philosophy for which the central tenet is that learners actively construct their own knowledge rather than passively receive it from external sources such as teachers and textbooks. According to Barton et al., knowledge is "generated through exploration and participation in discussions," and the embodiment of this premise in New Zealand mathematics education coincides with that nation's "interest in the role given to culture" (p. 11).

Matthews (1995), the editor of the international journal, *Science and Education*, and formerly Professor of Science Education at the University of Auckland, has forcefully argued that constructivism in mathematics and science education does not have a strong and coherent philosophical base, and that therefore any attempts to develop successful education environments based on a constructivist philosophy are unlikely to be successful. Other writers (for example, Allchen, 1995; Kilpatrick, 1987), have also argued that although the foundations of constructivism (and of radical constructivism[13] in particular) are shaky, the passion and zeal of crusaders seeking to gain large influence over practice has been dangerous in the sense that unsuspecting "followers" have been easily manipulated.

Indeed, Black and Lucas (1993) have asked whether there could be "any great value in more researchers collecting more data about phenomena from more groups of children in more countries, rather than trying to fit the existing data into a more adequate theoretical framework?" (p. xii). Just *how* pressing it is to develop adequate interpretations of what constructivism might mean for mathematics classrooms in different cultural contexts depends on the version of constructivism embraced. As Allchen (1995) has pointed out, constructivism is, after all, "a broad church doctrine" (p. 303)—so broad in fact that Good, Wandersee, & St. John (1993) were able to distinguish between 21 varieties of constructivism. In mathematics education there is a predictable spread of positions from radical constructivism through to soft constructivism—in the latter, constructivism frequently just means good, innovative or engaging teaching, where children are respected and their ideas are given some attention or credence, and where teachers do not dominate classrooms.

Regrettably, in Western nations there are relatively few serious historians of mathematics education. This state of affairs has meant that many mathematics education reforms, such as the move towards constructivism during the period

13. Radical constructivism emphasises the epistemological bases for education which were expounded by Jean Piaget, the Genevan developmental psychologist. Its chief protagonists are probably Professors Ernst von Glasersfeld and Les P. Steffe.

1985–1995, and the worldwide acceptance and then rejection of the "New Math" over the period 1960–1975, were planned and implemented without any true appreciation of historical direction[14] by the reformers. It is hardly surprising that such reforms have been rudderless and have failed to respond adequately when difficult circumstances have arisen.

Because of the need for mathematics educators to gain a better understanding of the origins of contemporary mathematics education research themes and methods, a brief history of international mathematics education research is provided in Chapters 3 and 4 of this book. In providing such a history we are fully conscious of the fact that there can be no such thing as objective history. The decision to focus on one theme rather than another, to link one event or document causally to another, and the various threads of argument which are used, must all be influenced by the backgrounds and priorities of those constructing historical text. Although the themes we develop in Chapters 3 and 4 reflect our own priorities and interpretations, we make no apology for attempting to provide an historical perspective which will enable readers in the Asia-Pacific region to place contemporary mathematics education research movements within a wider time frame.

In the 1990s the need to place mathematics education developments in their historical contexts is being increasingly recognised in Asia-Pacific nations. Thus, for example, a recent Chinese publication entitled "A guide to the research on mathematical education" (Zhang, 1994) contains two articles, written in Mandarin, on the history of mathematics education in China, and Lim (1995) has recently written an account of the history of mathematics curriculum development in Malaysia. Barton et al. (1995) have also provided helpful commentary on the history of mathematics education in New Zealand/Aotearoa, paying special attention to the history of Maori mathematics education.

Illuminative Evaluation Case Studies can be Important

The value of the case study, in which a particular set of events, or project, or organisation is studied in depth for the purpose of identifying principles of operation which might throw light on ("illuminate") other related sets of events or organisations, has increasingly been recognised over the past two decades. The turning point in the acceptance of "documented stories" as useful research by the mathematics education research community probably came in the form of the seminal study by Stanley Erlwanger (1975) into the effects of a mastery learning program on the teaching and learning of mathematics in an elementary school in the United States of America.

Many mathematics educators who have read the report of Erlwanger's study have been impressed, not only by the apparent reality of what was described, but also by the insights it provided into mastery learning. Those reading Erlwanger's report became acutely aware of the potential pitfalls of mastery programs in mathematics education. Aspects of mastery programs which they had experienced

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14. Bossé (1995)and Kilpatrick (1992), Jahnke (1994) and Usiskin (1994a,b) are among those who have recently written significant articles on the history of mathematics education. Both authors have also contributed to the field—see, for example, Ellerton and Clements (1988, 1994), Clements (1992), and Clements, Grimison, and Ellerton (1989).

but which had remained outside the level of consciousness, were suddenly recognised. Yet, Erlwanger did not use inferential statistics.

Erlwanger was supervised by Dr Jack Easley, of the University of Illinois. It was Easley, often working with Bob Stake, also of the University of Illinois, who championed the illuminative case-study approach to education research during the 1970s and 1980s. Stake described this approach as responsive, qualitative, and interpretive in which there is a heavy reliance on direct personal observations and personal interviews. According to Stake et al. (1994):

> Our values about education policy and practice have developed over the years and we do not try to hide them in our reports. Our reports are carried by our interpretations, drawn from our experience and points of view. In some instances, to some people, they appear impressionistic, and they are, but if we have done the work well, they are thoroughly worked impressions. It is the intent in qualitative research to makes assertions rooted in prior comprehension as well as data. (pp. 16–17)

The difference between this and other forms of interpretive research is that the evaluation team unashamedly seeks to understand and report the meanings of structures and events in terms of their own research backgrounds, scholarship and professional experience.

A report—referred to earlier in this chapter—of an illuminative evaluation directed by Stake, of a Mathematics and Science teacher professional development program conducted by the Teachers Academy for Mathematics and Science (the "Academy") in Chicago in the early 1990s, can be taken as illustrative of the nature of illuminative evaluations, and of how they are reported. The original proposal for funding submitted by Stake contained eight "foreshadowing" issue questions (see Stake et al. 1994). These were large questions (like, for example, "How has teacher knowledge, pedagogy, and belief been affected by participation in Academy activities?" and "Do present student achievement testing and assessment complement or deflect from Academy efforts?").

However, the evaluation team, which comprised eight people, stipulated in the proposal that in line with its "responsive" style of illuminative evaluation, and in response to observations in schools and at the Academy, these questions could be refined, new questions asked, and priorities changed during the conduct of the evaluation. In fact, the set of issue questions did change as the study progressed, with the number of questions being increased to 15. Some of these included: "Were the schools and teachers most needing help the ones participating?" and "Was district emphasis on standardized achievement testing consistent with Academy emphasis on TIMS and MathTools?"

The evaluation team's report, a 120-page document titled "The burden," provided a powerful indictment of the negative effects on schools and teachers of (a) centralising standardised testing programs; (b) the arbitrary, external, setting of standards; and (c) the detailed prescription of curriculum outcomes (Stake et al., 1994). Those writing the report used everyday language, and the report itself resembled a story told "warts and all." The title of the report derives from the following passage, which summarises the main finding of the evaluation study, and illustrates the style of writing:

> The load these schools can bear is not adequately being assessed. Unrealistically, the State, the reform movement, the assistance organizations have been raising the load. They are not helping school people decide, given the resources available, what responsibilities to diminish. Part of the burden is those long lists of state, national, community, and professional goals and standards, idealistic views of what ought to be. Voltaire's words were "The best is the enemy of the good." Some schools like Westlake have been getting better gradually but far, far too slowly. Long term failure is almost certain, partly because the outsiders, as well as the Westlake LSC [Local School Council], have too grand a view of what they should be. The vision of democracy-in-action to effect school reform is not possible until committees are setting the goals as well as the programs. (pp. 84-85)

The evaluation team's report is full of statements like the one which follows (below). Although the report is about schools and a professional development program in a district in Chicago, the reader is almost compelled to reflect on the health of mathematics education, and indeed on all aspects of schooling, in general. What are the boundaries of legitimate research? Illuminative research reports, such as "The Burden," present data which more or less demand attention. The unasked question is always: "How much can this be generalised?"

> Markers of success that fitted the school's circumstances and met external standards at the same time were hard for students and teachers to identify. Teachers believed that classroom order was a prerequisite for teaching. The children seemed almost universally unable to monitor their own behavior. Teachers responded to poor impulse by increasing external behavioral controls. Children responded to behavioral control in turn with more negative behavior. I observed an experienced upper-math teacher who prided herself on careful planning and close management of classroom flow. In one of her math classes, 50% of the teacher's comments were directed toward behavior rather than curriculum. With the "good" group she was able to devote more class time to instructional comments. In both groups, the teacher's activity level and perceived responsibility for learning outcomes was far greater than the students'. By the end of her work day, she looked exhausted and angry, and the students as a group seemed no closer to valuing the acquisition of math skills. A newer teacher said, "I start the day with energy, but by 1 pm. I'm shaking inside because everything is such a struggle." (p. 53)

Stake (1988) has argued that case studies, even of unique and perhaps "unrepresentative situations," help readers to generalise, and in that sense can be illuminative for teachers and administrators in many countries. This is particularly likely to be true if several reports are written by members of a team of evaluators who investigate different cases, but have agreed on a common set of issues which will need to be considered in each case. This methodology was used in a co-ordinated report on a set of nine case studies of National Science Foundation (NSF)-funded middle-school science and mathematics teacher preparation programs (Stake et al., 1993).

Interestingly, Stake et al. (1993) concluded that the nine models of science and mathematics teacher preparation that they investigated remained local, not general models or models suitable for adoption elsewhere. They indicated that both the NSF staff and those involved in the teacher preparation programs had been

"largely committed to the same localist, non-centrist ethic" (p. 301).Those involved in the teacher preparation programs had had to choose "between paramount commitment to the development of a national collection of valid models of teacher training or to make their one program work on their own campus" (p. 301). Almost without exception, they chose the latter. From a research methodology perspective, the case approach adopted by Stake et al. (1993) directly raised the question of whether the NSF should have been more systematic in its allocation of funds. And, according to Stake et al. (1993), as a result of reflection on this issue, NSF had subsequently moved to more systemic support of educational reform.

Other Forms of Education Research can be Important

There are many categories of education research which have not been mentioned thus far into which investigations in mathematics education issues could be classified. Labels such as "critical," "phenomenological," "postmodern," poststucturalist," and "feminist" are used. Most of these labels are of recent origin, and are sometimes referred to as belonging to the "postpositivist enquiry" era of education research (see Lather, 1991).

Scholars who identify themselves as postpositivist researchers tend to use the pejorative term "positivism" to refer to the dominant modes of social science enquiry, which they see as "a code of meaning, at best 'bourgeois' and at worst 'reactionary,' and supporting the status quo" (McCormack, 1989, p. 20). According to Lather (1991):

> Too often, the denigration of positivism equates contemporary positivisms with the Cartesian version of objective knowledge which some argue has long been left behind ... The adequacy of contemporary positivisms in terms of philosophical assumptions and practices of deductive logic, hypothesis testing, operational (measurable) definitions, and mathematised language is much debated ... My argument ... is not so much against such practices as against their hegemonic status in the doing of social science, their status as "the" scientific method. (p. 9)

For an overview of these contemporary research methodologies, the reader is referred to Lather (1991); a more detailed treatment is beyond the scope of this book. It should be noted, however, that an increasing amount of mathematics education research is being carried out under the postpositivist banner (see, for example, McBride, 1989; Skovsmose, 1994a,b; Walkerdine, 1990; Zevenbergen, 1994).

Mathematics Education Research in the Asia-Pacific Region: Structural and Organisational Framework

In view of the fact that space does not allow us to present a detailed literature review covering all aspects of mathematics education research in the Asia-Pacific region, it will be useful here to provide a brief summary of some—though certainly not all—of the organisations and structures within the Asia-Pacific region which

have tended to support mathematics education research. Some knowledge of these organisations and groups should assist readers to understand better the forces which have produced a gradual development in the mathematics education research culture in the region over the past three decades.

Earlier in this chapter we argued that, given the emphasis placed on the importance of mathematical literacy, there is a universal need to dedicate much more funding and resources to support mathematics education research. This need is particularly evident in the Asia-Pacific region, where many of the rapidly developing education systems are faced with difficult problems associated with mathematics curricula, and with the need to change unsatisfactory approaches to the teaching and learning of the subject.

Regional Centre for Education in Science and Mathematics (RECSAM)

Commitment to mathematics education research is surprisingly strong in the region. For example, the Ministers of Education of nine Southeast Asian nations (Brunei Darussalam, Cambodia, Indonesia, Lao PDR, Malaysia, Singapore, Thailand, The Philippines, and Vietnam) are committed to maintaining and supporting the Regional Centre for Education in Science and Mathematics (RECSAM) which was established in Penang (Malaysia) in 1967, and is administered by the Southeast Asian Ministers of Education Organisation (SEAMEO), in co-operation with the Malaysian government.

RECSAM offers professional development and research programs for leading educators from all nine contributing nations (Sulaiman & Remorin, 1993). Since 1978, it has produced the *Journal of Science and Mathematics Education in Southeast Asia*, a high quality research journal. It has published numerous research reports and monographs, and has provided leadership in the area of action research (see, for example, the *Technical Reports* of the *RECSAM Teaching—Learning Project* (Leong & Ferrer, 1991), and the reports of the *Thinking in Science and Mathematics (TISM) Project* (Tan & Sim, 1991). RECSAM also hosts research conferences and workshops. For example, in February 1992, RECSAM hosted an international conference on the theme *State of the Art of Research in Science and Mathematics Education in Southeast Asia and the Pacific*. Sponsors included the Australian and Canadian Governments.

Other Mathematics and Science Education Centres

Many universities in the Asia-Pacific region have established regional mathematics and science education centres (for example, the Institute for Science and Mathematics Education Development at the University of the Philippines and the Science and Mathematics Education Centre at Curtin University of Technology, Perth). In Papua New Guinea, the University of Technology's Mathematics Education Centre published a comprehensive series of research reports (see, for example, Clarkson, 1984), and held annual research conferences, before the Centre closed in the mid 1980s.These research reports provided a unique commentary on forces influencing mathematics learning in a country which has about 750 languages and an equal number of indigenous counting systems. The counting systems have been extensively studied and documented in a seminal doctoral study by Lean (1992), and in the 24-volume appendix to this study (Lean, 1995).

Most national Ministries of Education appoint dedicated mathematics education personnel—such as are located within the Institute for the Promotion of Teaching Science and Technology (IPST) in Bangkok. In most cases, however, their work is largely concerned with curriculum development, including text book and test development, rather than on research.

Mathematics Education Research Group of Australasia

The Mathematics Education Research Group of Australasia (MERGA) was established in 1976,[15] and its 19th annual conference was held in Melbourne in 1996. MERGA, with a membership in 1996 of over 300 researchers, publishes a four-yearly review of mathematics education research carried out in Australia, New Zealand, and Papua New Guinea. It also publishes the *Mathematics Education Research Journal*, an international refereed research journal.

Reviews of Mathematics Education Research

Several overviews of research carried out in various nations of the Asia-Pacific region, and publications containing research abstracts have been produced. For example, the Southeast Asian Research Review and Advisory Group (SEARRAG),[16] has published a number of research reviews, including *State-of-the-Art Review of Research in Mathematics Education in Malaysia* (Lai & Loo, 1992).

RECSAM has produced a *Summary of Research Reports for the Years 1986–1992* (Roadrangka & Liau Tet Loke, 1993), as well as *Research Abstracts, 1980–1990* (RECSAM, 1991).[17] Research monographs produced by the Centre for Applied Research in Education at Nanyang Technological University include reports on mathematics education research (for example, Cheung, Koh, Soh, & Mooi, 1990; Cheung, Mooi & Loh, 1991).

MERGA has published several comprehensive reviews of mathematics education research in Australasia.[18] In addition, the Australian Department of Employment, Education and Training funded the "Asian Perspectives on Mathematics Education Project," and in 1994 and 1995, this Project, in conjunction with the International Commission on Mathematical Instruction (ICMI), sponsored

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15. The first annual MERGA conference was held at Monash University (Melbourne) in 1977. Originally, MERGA was known as the Mathematics Education Research Group of Australia.

16. SEARRAG is "a consortium of individuals from countries in the ASEAN region who are concerned about, and involved in, increasing the contribution of educational research to educational policy and practice in the region" (Aziz & Ghani, 1992, preface).

17. Together, these provide a summary of all of the research undertaken in research--based RECSAM Diploma Courses during the period 1980–1992. Commentaries and tables which provide an overview of the major research themes are also provided.

18. In particular, MERGA has published a number of four-yearly reviews (see, for example, the collections edited by Atweh and Watson (1992), and by Atweh, Owens and Sullivan (1996). MERGA plans to continue to publish these reviews.

the publication of two reviews on mathematics education in Asia and the Pacific (Bell, 1993, 1995). ICMI also sponsored a regional conference, held at Monash University in 1995, with the theme "Regional Collaboration in Mathematics Education." This was attended by almost 400 international mathematics educators, the large majority of whom were from the Asia-Pacific region. A book of conference proceedings, consisting of over 850 pages of research reports and summaries of thematic presentations, was published (Hunting, FitzSimons, Clarkson & Bishop, 1995).

Increasing Regional Collaboration in Mathematics Education

Clearly, over the past decade, there have been attempts within the region to establish and maintain networks between mathematics education researchers. An International Study Group on Mathematics Education in Eastern Asia met at the 7th International Congress on Mathematical Education (ICME-7) in Quebec in 1992. The working languages of the meeting were Chinese, Japanese, Korean and English, and one of the main topics discussed was how greater communication and cooperation could be facilitated among mathematics educators in the Asia-Pacific region.

ICMI, UNESCO, and the Southeast Asian Mathematical Society have combined to support biennial conferences.[19] In 1994, ICMI sponsored a China Regional Conference on Mathematical Education which was held at the East China Normal University, and featured keynote speakers from many countries.

Purposes of the Present Book

The above commentary on mathematics education activity in the region is not comprehensive. That is because there is no existing summary of the work of institutions or organisations involved in mathematics education in the region. However, over the past decade, there has been a heightening of regional consciousness concerning the importance of mathematics education and the need to establish networks between those who work in mathematics education.

It is our conviction that there is a particular need to establish links between mathematics education *researchers* in the Asia-Pacific region.

This book will acquaint readers with contemporary directions and methodological issues in mathematics education research, and will relate these to mathematics education practices, possibilities, and assumptions in the Asia-Pacific region. The book will also seek, in an uncompromising way, to identify and elaborate the kind of research which, from our perspective, is needed to improve mathematics education in the Asia-Pacific region. Towards that end, ten recommendations concerning the role and potential of mathematics education in the region will be made.

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19. The 6th Conference was held at Surabaya, East Java, in June 1993, and addressed the theme: *Facing the Challenge of Future Mathematics Education*.

In a book such as this, space is limited and therefore our arguments will not always be fully documented. Despite the lack of detail, we hope that the overview provided will encourage and facilitate communication, cooperation and exchange between researchers in the region.

We are conscious of the fact that, since both of us are Australian mathematics educators, those from other nations in the region may feel that their voices will not be heard in this book. Although we have worked extensively throughout the region, we recognise that it will not be possible to present all of the achievements, aspirations and needs of each nation's mathematics teachers, educators and education systems. We do hope, though, that one outcome of this book will be that those who teach mathematics in the region and those who regard themselves as mathematics educators and mathematics education researchers will realise that they have different but complementary knowledge, understandings, skills, and opportunities.

Policy formulation in the area of mathematics education should not be left in the hands of education "experts" who neither know the mathematics education literatures nor have recent experience in the teaching of mathematics or mathematics education. Similarly, decisions with regard to agendas for mathematics education research needed in the region should not be made by those (a) who have never themselves conducted mathematics education research; and (b) are clearly outside the international mathematics education research community. If the region's mathematics educators—where, on this occasion, that term is used broadly—are able to work together, to pool their combined expertise and their knowledge of forces influencing the directions in mathematics education, then the chances of achieving improvements in the quality of mathematics learning in the region will be greatly enhanced.

2 What are "the Basics" in Mathematics Education?

Recently, those controlling mathematics curricula in the United Kingdom decided to downplay the role of the "using and applying" strand of the national curriculum, and to make the teaching of long division algorithms a legal requirement for all government schools. As Noss (1994) has pointed out, it would be difficult to sustain any argument that helping all students to use and apply mathematics should *not* be a basic concern of any mathematics curriculum. It is equally apparent that in the 1990's knowing how to divide a three-digit number by a two-digit number is no longer a basic skill—given the ready availability of inexpensive hand-held calculators and the fact that few people find the need to do such calculations more than once or twice a year. As Noss (1994) has stated, such knowledge is, for most people, "both anachronistic and *use*–less" (p. 45).

This long division episode draws attention to a much larger issue which is of fundamental importance to mathematics educators and mathematics education researchers. Issues concerned with who needs what, so far as mathematics education is concerned, are not always resolved on purely logical grounds. Wranglings over curriculum are inevitably value-laden and politically determined exercises[20] (Damerow & Westbury, 1985). Someone, or some group of persons, decides who needs what. This raises the matter of the politics of mathematics education, and how this might relate to mathematics education research.

One thing is clear: "the basics" in mathematics education—whatever they might be—are not independent of culture, and even within a single culture they vary with time. Table 1, adapted from Reys, Reys, and McIntosh (1995) and from Shumway (1994), is based on computational tools used by adults in the past, present and future. Although the table's implicit message is important, we would add the cautionary note that the same patterns are unlikely to apply in all contexts.

20. Although, by the word "politically" we do not mean to imply that the decisions are necessarily made in a parliament or in a parliamentary committee.

Table 1
Percentage of Time Allocated to Various Types of Computations

Type of Computation	Adult Usage	Elementary School Experiences *Before 1975*	*Now*	*Future*
Written computation	5%	85%	70%	20%
Mental computation	30%	10%	10%	30%
Estimation	30%	5%	10%	20%
Calculators	35%	0%	10%	30%

Research carried out in Australia (McIntosh, Bana, & Farrell, 1995), and in Japan and the United States of America (Reys & Nohda, 1994), indicates that although students saw mental computation as being more important than written computation, more class time is dedicated to the latter. That is likely to be the case in most nations in the Asia-Pacific region. Curriculum change is needed, so that more class time will be spent on assisting all students (that is to say, future adults) to become fluent, competent and confident with the four computational forms listed in Table 1 (see Reys et al., 1995: Sowder, 1992; Willis, 1988/1989).

Similar comments could be made in regard to the need to update many secondary school mathematics curricula in the region, as a result of the increasing availability and impact of graphics calculators and advanced algebraic, statistical, and geometrical computer software packages (Penglase & Arnold, 1996). Yet, in most schools in the region, access to such resources is unthinkable. Clearly, mathematics education reformers and curriculum developers in the region have a difficult balancing act to perform.

The Politics of Mathematics Education

During the last decade mathematics educators have increasingly realised that mathematics education is part of education and, therefore, is political because it is concerned with access to power and privilege (Brousseau, 1984; Noss, 1994; Volmink, 1994). However, it is possible that many educators have failed to recognise that all mathematics education inevitably carries subtle hidden messages about how elements of culture and society should relate. As Frankenstein (1995) has pointed out, everyone can recognise political overtones associated with questions in mathematics textbooks in which guerilla fighters are portrayed as helping peasants, and landlords as demanding compound interest from poor peasants. However:

> Even trivial maths applications, like finding the total from a grocery bill, carry the non-neutral hidden message that it's natural to distribute food according to individual payment. Even traditional maths courses which provide no real-life data carry the non-neutral message that learning maths is separate from helping people understand and control the world. (p. 168)

erestingly, Apple (1995) commented that he would like to make the chapter by

Frankenstein from which this last quotation came "required reading for all mathematics educators and researchers" (p. 341).

Mathematics education research is political because it cannot operate outside of the politics of mathematics education. This realisation has motivated mathematics education researchers to attempt to lay bare the political dimensions of mathematics education. It is not uncommon to find statements such as the following appearing in texts dedicated to mathematics education:

> I have turned to the history of mathematics in the hope of finding some evidence that mathematics can be an activity for the masses, by the masses. But this quest has left me comfortless. No matter where I look, I can only find confirmation that only the élite, the select few were and still are involved in mathematics. Now, with the advent of ethnomathematics came also a recognition that the monolithic voice of Euro-centric mathematics cannot be left unchallenged. Ethnomathematics is therefore one of the voices that has come up in revolt against the suppression, lack of recognition and the exclusion of the mathematical ideas of other cultures. Yet strangely, when I appeal to the literature on ethnomathematics, I see again only the rich, the influential, the powerful, the privileged, having direct access to and control of mathematical ideas, in their own cultural contexts. (Volmink, 1994, p. 57)

Thomas (1992) has argued that the politics of mathematics education has two dimensions: first, there is the social context, which takes into account factors such as language, culture, gender, socio-economic-status, and access to technology; and second, there are government policies and practices which have an impact on how such research is funded.

In this section we shall take into account both of these dimensions, but not necessarily separately—in fact, we shall be especially concerned with the intersection of the two. The brief case study presented below illustrates the two dimensions which Thomas (1992) identified.

Round Pegs in Square Holes

In 1980 one of the authors (Clements) worked in the Mathematics Department of the Papua New Guinea University of Technology (PNGUT) in Lae. During that time he and a colleague, Glen Lean, designed and carried out research into factors influencing mathematics education in Papua New Guinea. They travelled to many parts of Papua New Guinea, and it did not take them long to realise that teachers of mathematics in Papua New Guinea Community Schools were being asked to fit round pegs into square holes.

Lean and Clements's first "research expedition" took them into the Eastern Highlands.[21] There, they visited the first government community school which had ever existed in a village. The school buildings consisted of two small thatched-roof rooms which had been built the year before. During the time they were present at the school they found 120 children crowded into two rooms—boys in one room, girls in the other. The one teacher was expected to go from room to room.

21. See Clements and Lean's (1981) report on this and three other mathematics education "research expeditions" in Papua New Guinea during 1980.

This young teacher was working under difficult conditions: he had charge of 120 pupils in two rooms; he did not speak the first language of the pupils, and they did not speak his first language; the language of instruction was English,[22] but the young teacher spoke English only haltingly. Furthermore, no other "local" spoke English, and there was little opportunity for him to improve his fluency in the language. To make matters worse, he had no secretarial or technical support, hardly any specialised teaching equipment, no tap water, no electricity, and no radio or television.

The teacher was a recent Teachers College graduate who had received much instruction during his College years in the use of the methods and equipment advocated by Zoltan P. Dienes, whose ideas had been officially adopted by PNG education officials, sometime in the 1960s. The day Clements and Lean were present the teacher delivered a "Dienes lesson." He used a set of wooden attribute blocks (which, in Dienes programs, were used in a wide range of activities).

Thinking in terms of concepts like circles, squares, triangles, rectangles, and diamonds was not common in the culture from which the children came. Nevertheless, learning about such shapes, using attribute blocks, was a mandatory part of the national syllabus, and that was why the Dienes attribute blocks had been provided by the Education Department. During one of the lessons observed by Lean and Clements the teacher held up a large, red rectangle, and asked the children to chant after him, in English:

> This is a big red blocks.
> This big red blocks is an oblong ..."

It was clear to the observers that what was going on in the name of mathematics in that school was neither helpful nor useful for most of the children, who did not understand what the teacher was saying, or even what they themselves were chanting. Yet, this young, well-meaning teacher was doing exactly what he had been told to do in his teacher-education course; he was even using structured aids advocated by one of the world's leading mathematics educators. The conclusion was inescapable: those responsible for the mathematics education program in the Papua New Guinean Teachers College—mainly expatriate educators and bureaucrats—had used a curriculum (together with associated teaching methods) which was not in harmony with the cultures and future professional needs of prospective teachers (Hayter, 1982).

The reason for citing the above example is to suggest that, given the rapidly changing situations in many of the nations in the Asia-Pacific region, politicians and education bureaucrats are striving to identify "best practice" procedures for mathematics teaching and learning. However, contemporary mathematics education literatures suggest that there is no such thing as culture-free "best practice" in mathematics teaching and mathematics teacher-education programs (Bishop, 1990, 1992; Clements, 1995a).

It is not an exaggeration to say that what is appropriate and good for one group of learners may not be appropriate for another group of learners. Simplistic

22. By government policy, all classes had to be taught in English.

solutions, such as the imposition of uniform curricula, or textbooks, or mandated teacher education programs are unlikely to provide acceptable answers. For example, unless they have been carefully designed by enlightened educators, mandated core mathematics teacher education programs are likely to generate young teachers with élitist, separatist views on mathematics education—"élitist" because the programs derive from narrow views about the nature of mathematics, and "separatist" because mathematics is presented as separate rules and activities, not linked with each other, or with other curriculum areas, or with the learners' personal lives.

The discussion in this section has drawn attention to a cultural mismatch represented by an inexperienced teacher attempting to follow a Western mathematics curriculum at a school where this made little sense to learners. The episode raises the twin issues of culture and control: the teacher was teaching the children what he was *expected* to teach them, using methods that he had been trained to use in his initial teacher education program.

Despite the fact that he was the only teacher in the district, that only a handful of people living in the villages of the neighbourhood had ever attended a school, that the villagers could neither read nor write, and had little idea of what "normally" took place in schools, the teacher had felt constrained to do what was normally expected of teachers throughout the country. Like most other teachers in Papua New Guinea (and, of course, most other countries), he felt the need to follow the prescribed curriculum, to keep the children in his care under control and "on task," to introduce topics in as interesting way as possible, and to follow recommendations from "experts" on how mathematics is best taught. Textbooks supplied to his school by the Papua New Guinea Ministry of Education were an invaluable aid, as were notes taken in his "Mathematics Methods" course during his pre-service teaching course. Because of government policy—and, possibly, the fact that he could not speak the first language of his pupils—he used English to introduce and explain mathematical concepts.

The decision to teach in the way he did had been an *internal* one, arising out of his desire to act in as professionally responsible manner as possible. However, despite his geographical and professional isolation, he also knew that from time to time there would be *external* checks on his teaching performance—these would come in the form of reports written by government "inspectors," who would visit his school, examine his school records, observe his teaching and, perhaps, test the children.[23]

The young teacher did not question the relevance of the mathematics curriculum he was following, or wonder too much about the appropriateness of his teaching methods. He was attempting to cope with a difficult situation by doing what he thought any responsible young teacher would do—he was following the

23. With respect to testing, inspectors faced the following dilemma: they were required to test in English because this was government policy, but the students understood very little English. Even if, at a particular school, an inspector wanted to check the students' understanding by questioning them in their own language, this would not have been normally possible because the inspector would not have been able to write or speak the students' first language.

curriculum prescribed by his Ministry, and was using teaching methods and approaches which he had been taught in his pre-service teacher education program.

Berman (1992) has argued that networks and national and international aid agencies, linked to, and located within industrialised nations, have been an important means by which First World countries have controlled education in developing nations. Certainly, the young Papua New Guinea teacher in the above case had been trained to use Dienes' methods in a teacher education program largely designed by experts who were not PNG nationals. According to Berman (1992), aid agencies should actively foster locally-controlled, non-governmental organisations (NGOs) as the primary means by which so-called "Third World" nations develop their education systems in line with their own education needs.

Sacrificial Lambs

The folly of "outsiders," however well educated and well meaning, imposing fairly common mathematics curricula on students from different cultures has become increasingly evident. Such attempts inevitably result in learners from major groups (like, for example, females, the working classes, and certain ethnic and racial groups) being disadvantaged, yet this happens in a social climate aiming to achieve "equality of educational opportunity" (Clements, 1992). Surely it is important that politicians, education administrators, mathematics teachers and mathematics education researchers in all countries do not continue to condone procedures where a majority of students who are required to learn mathematics become sacrificial lambs at the twin altars of education efficiency and economic rationalism (Pitman, 1989).

Many people, and especially politicians, employers, and education administrators, believe that what goes on in a nation's school mathematics programs is intimately linked to the economic condition of that nation. Often powerful elements of communities call for back-to-the-basics approaches to school mathematics, supported by regular state- or nation-wide mathematics testing programs, as a means of ensuring that mathematics teachers will be publicly accountable for their actions. A problem can arise if these powerful figures think they know more about mathematics education than they really do, and are unwilling to be confronted by research results which suggest that back-to-the-basics, behaviourist approaches, supported by externally prescribed curricula and heavy external assessment programs, will not generate better mathematical understanding in learners.

The point we are making was amplified in a large-scale investigation into the literacy and numeracy skills of Australian children aged 10 and 14 years, carried out by the Australian Council for Educational Research (ACER) in 1975. The large sample for this study included subsamples of Aboriginal students living in remote parts of Australia, mainly in the Northern Territory, and urban Aboriginal students from different parts of Australia. The urban Aboriginal students performed significantly less well on the numeration tests than Western children of the same age, and the Aboriginal children living in remote regions showed next to no understanding of the written tasks.

Some data from the 1975 ACER study will serve to illustrate the point we are making. Both the 10- and 14-year-old students who were tested were asked to write down the time shown on a watch-face: the watch used for the younger students had Arabic numerals on its face, but that used for the older students had only strokes. The times shown were 11:35 and 4:40 for the 10- and 14-year-olds, respectively, and results obtained are set out in Table 2.

Table 2
Performance on Time-Telling Tasks by Three Groups of Students

Age Group	% of Correct Responses for ...		
	Remote	Urban	Overall Australian
10 year-olds	2	25	71
14 year-olds	3	73	89

Bourke and Parkin (1977), in reporting these findings, concluded that the questions on the literacy and numeracy tests involved ideas which were largely foreign to the cultures of the Aboriginal children living in remote regions, a point emphasised by Pam Harris (1987, 1989, 1991) and Helen Watson (1988, 1989). Of course, such a conclusion came as no surprise to those Aborigines who live in remote regions and to others with an empathy for Aboriginal cultures.

Bourke and Parkin (1977), in commenting on the very small percentage of Aboriginal respondents from remote communities who gave correct answers to these questions on time-telling, stated that the tasks themselves were "certainly outside the experience of many" (p. 149). Harris's (1991) book on Aboriginal time, space, and money concepts lends strong support to this view.

The question remains whether such knowledge is "basic," even for the Aboriginal children in remote communities. In future years most children—and possibly all—living in remote communities will find themselves in situations and locations where knowledge of Western time concepts and measuring devices, is important. If and when this occurs they will be disadvantaged if they cannot "tell the time" (in the Western sense of this expression).

The point is that what is regarded as "basic" in one culture at a given time can be irrelevant in another at that time, but the degrees of relevance could change within a few years. Certainly, the existence of a national curriculum in which necessary outcomes of learning for students are prescribed is likely to be sufficient to ensure that many teachers will waste much time trying to help unprepared and increasingly disaffected students to acquire skills which, although described as "basic," are, for these learners, almost meaningless. But who is to say that the skills will be irrelevant within a few years?

Who Should Make Mathematics Curriculum Decisions?

Adler (1994), in the course of identifying and addressing issues associated with mathematics education in post-Apatheid South Africa, has spoken of the need for teachers of all ethnic backgrounds in South Africa to be democratically involved in the formulation of policy. "Issues such as who participates, what knowledge is

selected, by whom and how it is transmitted and how it is evaluated" as well as class and gender, all need to be brought to the forefront (p. 105). We are certain that such issues are pertinent not just in South Africa, but in all nations and particularly, perhaps, in the nations of the Asia-Pacific region.

Historically, according to a UNESCO Report entitled *Mathematics For All* (Damerow, Dunkley, Nebres, & Werry, 1984), mathematics curricula were developed for an élite group of students who were expected to continue their mathematical and scientific studies in tertiary institutions (on this, see also Ellerton and Clements, 1988). But with more and more children attending schools on a regular basis, students from less selective backgrounds, and with different vocational aspirations and daily life requirements, have entered the education systems in greater numbers. These students have often found existing mathematics curricula to be unduly abstract, impractical, and irrelevant.

The UNESCO Report lamented the fact that old élitist curricula have frequently been transferred to developing and third-world countries where different social and cultural traditions have emphasised their inappropriateness (Damerow et al., 1984). There is increasingly strong evidence that children in non-Western cultures often think about measurement concepts, and about numerical and spatial relationships, differently from those in Western cultures (Clements & Del Campo, 1990; Harris, 1991; Hunting & Sharpley, 1988; Wheatley & Bebout, 1990).

Mellin-Olsen (1987) asked his readers to consider what would happen in mathematics education in a nation if it were not infiltrated by European educationists and publishers. He pointed to two mutually exclusive possibilities:

> A. It could stick to its traditions and original culture, and base its production on farming and scattered populations.
>
> B. It could aim at industry and technology in order to obtain the material standards set by Western measures.

He then asked:

> So what then about the choice of curriculum? Is it in the hands of the academics or not? Is it the psychologist or the anthropologist who really makes the decisions if their advice is followed? Whether yes or no the result is a political result. (p. 129)

Mellin-Olsen went on to argue that curriculum decisions should not be made by "expert" outsiders, but by the people themselves, for "it is really their decision, and this decision is related to a much wider context than that embedded in the walls of the educational institution: it is related to the context of society" (p. 129).

We believe that Mellin-Olsen did not go far enough: we would want to know exactly who he meant when he used the expression "the people themselves" (who, he said, should make the mathematics curriculum decisions). If, in the end, mathematics curriculum decisions are made by politicians or education administrators who are prepared to sacrifice the immediate needs of the majority of students in order to institute and support programs which history, and education research, suggest are doomed to failure then perhaps "the people themselves" will not make representative or wise decisions. As Mellin-Olsen himself made clear, mathematics education is a far more political arena than most people imagine.

Language and Garma

Jones, Kershaw and Sparrow (1995) have drawn attention to the essential differences between "Western" mathematics as represented in the curriculum of the Northern Territory Department of Education in Australia and "Garma" mathematics—Garma is a Yolngu metaphor for both an open meeting place and a process. Jones et al. (1995) quote extensively from Batchelor College documents[24] in which it is made clear that language and culture are vitally important aspects of Garma mathematics. According to Jones et al. (1995):

> Fundamentally different world views such as those just described translate equally dissociate language structure and function and vice versa. ... English is not structured for the purpose, whereas relevant Aboriginal dialects are. In a very real sense culture *is* language. (p. 19)

Jones et al. (1995) refer extensively to the work of Christie (1994) and Stephen Harris (1990) in elaborating the view that although Western mathematical knowledge is essentially antithetical to Garma mathematics, there are aspects of contemporary thinking about Western education, such as cooperative group learning, which also have an important place in a Garma approach to mathematics education.

The fundamental importance of cultural aspects of education has been highlighted in unequivocal terms by Stephen Harris (1990):

> The nature and degree of the difference between Aboriginal and European culture is so great that the only honest conclusion we can arrive at is that they are largely incompatible. The two cultures are antithetic—consisting of more opposites than similarities. They are warring against each other at their foundations. Recognising and accepting the truth of the term incompatible was for me ... the point of theoretical liberation and the starting point for a more effective educational theory to be applied in Aboriginal schools. This degree of difference is so great that it is harder to find what they have in common in cultural terms than it is to see the differences. (p. 9)

If Harris's view is accepted, and if in fact language is intimately related to culture, then there would appear to be massive implications for those responsible for curriculum development in general, and for mathematics curriculum development in particular.

As Jones et al. (1995) have commented, the process whereby Aboriginal and other non-Western "minority" groups will have the right for their children to experience a curriculum which links with their personal worlds rather than being in opposition to it, is underway. Essentially, "cultural reaffirmation has translated into a decolonising process, particularly with respect to education" (p. 2). There can be little doubt that Western mathematics, together with its associated language and symbol forms, represents a highly specialised way of thinking, and to a certain extent that explains Bechervaise's (1992) description of mathematics as a foreign language.

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24. Batchelor College is a tertiary institution located near Darwin in the Northern Territory (Australia); the College prepares Aboriginal students to become teachers.

Yet, the reality is that most Aboriginal children and, more generally, children from other so-called "minority" groups, are required to attend school and learn a Western style of mathematics. Learning the language and symbol systems of Western mathematics is an essential component of the enculturation (some may say indoctrination) process. Harris (1992), who served as a mathematics curriculum specialist in the Northern Territory of Australia, made the point well in talking about "school shapes." She stated:

> In the Space Strand Investigation Guide I asked specific questions about what I called "school shapes." My labelling of them as "school shapes" goes back to my early days of teaching at Yuendumu when I was required to use attribute blocks for teaching maths. I absolutely hated using attribute blocks. It seemed to me quite nonsensical to sit there asking Warlpiri children "Give me a square, show me a circle, can you find a triangle?" and so on. I just thought that if we had to classify, we should classify something that was meaningful to them.
>
> ... In response to the Space Investigation Guide questions about school shapes, one linguist said that an Iwaidja woman (on Croker Island off the coast of Arnhem Land) described the plane shapes drawn by the linguist as follows: A square was described as "flat, smooth"; a circle as "round"; ... an oblong simply as "long"; and a triangle as "having sides."
>
> ... What I found intriguing was that from a linguist with Gurindji people in the desert ... came categories for solid shapes which were almost the same as those the Iwaidja woman gave for the plane shapes. They said that there seemed to be roughly three categories for solid shapes—flat, round and long. Books, mangoes, and boxes are "flat," berries, eggs, and balls would be in the category of "round," and yams, swags, and tin cans would be considered "long." It was noted that most things that would be considered "long" also had a component of being somewhat cylindrical. (p. 65)

One of the challenges facing mathematics education researchers is to explore further links between language, culture and mathematics education, and to make sure that their findings influence curriculum decisions. It is our view that in many nations this aspect has not been given enough attention by those who develop mathematics curriculum frameworks or "standards" documents.

Cultural Imperialism

Bishop (1993) argued that the fact that mathematics is not language-free has profound implications. After pointing out that the original language base of mathematics is probably an amalgam of Indian-Greek-Arabic-Latin influences, he added that from this base the Italian, Spanish, French, German and English developed their mathematics language repertoires over the past 400 years. At present, English is probably the principal medium for international mathematics research developments, and yet the indigenous languages of many groups in Australia, Papua New Guinea, Oceania, rural Africa, and tribal groups, which can be found in most nations in Southeast Asia, have language registers which are distant from those of English, or for that matter, from any of the other languages mentioned above.

Because there is such a strong link between culture and language, mathematics curricula imposed on minority groups can be regarded as representing a form of cultural imperialism. Although it is certainly true that not all languages express Western mathematical concepts in an efficient way, it is equally true that many of the concepts in the indigenous languages and cultures would not be easily expressed in English or in other languages spoken by large numbers of people. As Bishop (1993) has stated "If countries and societies within countries are to engage in the process of cultural reconstruction then the language element in relation to informal, non-formal, and formal mathematics education is critical" (p. 20).

There is a real danger that such is the power and prestige of Mathematics that, in the Asia-Pacific region, and in fact around the world, well-meaning leaders of society have foisted on unsuspecting school children a type of mathematics learning and content which many were not ready, both from the cultural and cognitive points of view, to learn. This has created so-called "minority" students who are regarded as "disadvantaged" (Secada, 1988, pp. 48-49). It is likely that in the Asia-Pacific region, as in other parts of the world, rigid streaming or setting policies, and traditional patterns of discourse in mathematics classrooms, have served to bolster false claims, to perpetuate myths about what is true and what is false, and to preserve racial, gendered, and social inequalities (Brown, Collins & Duguid, 1989; Leder, 1988; McBride, 1989; Mellin-Olsen, 1987; Perkins & Salomon, 1989; Popkewitz, 1988; Secada, 1988; Walkerdine, 1988; Wheatley & Bebout, 1990; Zevenbergen, 1993).

Most teachers of mathematics do not, of course, deliberately set out to teach in a way which will produce unequal outcomes. Nevertheless, it must be recognised that teachers—and parents, too—are themselves products of an education system which has generated inequity in mathematics education. It is difficult for them to imagine mathematics education environments which differ in important ways from those that they experienced when they were students themselves. Those responsible for developing and framing education policy need to co-operate with teachers, parents, mathematicians and mathematics education researchers in an attempt to reshape school mathematics curricula and pedagogy so that more equitable educational circumstances will prevail.

It could be the case that throughout the Asia-Pacific region, the net effect of up to ten to twelve years of compulsory mathematics instruction has been to convince most school leavers that they cannot do Mathematics (Ellerton & Clements, 1989a, p. vii). The internationalised version of mathematics—Bishop (1988) calls this "Mathematics," with a capital M—is being presented in schools around the world as if it were a form of external, objective, knowledge that "bright" students will acquire if they apply themselves diligently. It is accepted as part of the culture of schooling in many countries that "other," not-so-bright students will gradually fall by the wayside, mathematically speaking, although teachers hope that these students will have acquired enough of the powerful "objective" knowledge to be able to survive with dignity in their society. Yet, for many learners there can be no doubt that attempts to root "tomorrow's knowledge in the knowledge of yesteryear" (Mellin-Olsen, 1987, p. 131) have been inadequate.

One is reminded of Paulo Freire's (1985) words: "Propaganda, slogans, myths :e the instruments employed by the invader to achieve his objectives: to persuade

those invaded that they must be the objects of his action, that they must be the docile prisoners of his conquest." Freire added that "it is incumbent on the invader to destroy the character of the culture which has been invaded, nullify its form, and replace it with the by-products of the invading culture" (p. 114). Freire called for the establishment of dialogue which avoids cultural invasion, and dialogical manipulation or conquest.

Mathematics educators need to consider, carefully, the extent to which current practices and assumptions in school mathematics constitute, for many pupils, an invasion of culture. In a nation like Papua New Guinea, for example, with many of its 750 or so indigenous counting systems still surviving in the villages (Lean, 1992), a real tension has been created by the introduction into the Community Schools of a form of Mathematics that simply does not fit. It is true that school systems in many countries produce a small number of education survivors who proceed to higher educational studies. But, meanwhile, what has the education system done for the vast majority who exit from mathematical study believing not only that they cannot do the subject, but also that they never will be able to do it (Clements & Jones, 1983; Wilson, 1992)?

Mathematics for the Minority

Any satisfactory history of mathematics education in different nations around the world over the past 200 years would not fail to draw attention to how those responsible for framing the curricula of primary schools have always regarded mathematics as important. 'Rithmetic has always been regarded as the third R, and the community has accepted without question the argument that all primary school children have a right to acquire those basic numerical and measurement skills which would fit them to survive with dignity in a world which demanded arithmetical competence (Ellerton & Clements, 1988).

At the secondary school level, few have ever questioned the key role of mathematics, although for much of the nineteenth century it was assumed that only boys needed to study algebra, geometry, trigonometry, or the calculus. This discrimination based on gender (Clements, 1979) was less important than another discrimination based on family background, and around 1900 less than one per cent—mostly males, and from wealthy families—of people around the world had received any form of secondary education (Ellerton & Clements, 1988). Thus, at the beginning of the twentieth century only a tiny proportion of people in Western and non-Western nations had ever studied mathematics beyond arithmetic, and of those who had, relatively few had proceeded to study higher mathematics at a university. A "mathematics for the minority" way of thinking had become commonplace among students and educators, and this was accepted, by and large, by almost all adults (Clements, 1992).

Throughout the nineteenth century, and for much of the twentieth century, school and university administrators of those developing nations which were regarded as "colonies," were under pressure to duplicate the mathematics curricula prescribed for corresponding education institutions in "advanced" Western nations, and to maintain standards, through examinations, at about the same level as those in "home" institutions. Typically, the content of the

mathematics studied in the secondary schools and universities of developing nations, and the mode of assessment, were almost identical to those in "mother" countries.

Few if any questioned the place of arithmetic in the curricula of primary schools, or of mathematics in the secondary schools. Furthermore, over time, the universities produced their own crop of mathematics graduates, and many of these became secondary teachers of mathematics, and even academic mathematicians at universities.

This pattern continued through until well into the twentieth century. Those children who attended primary schools studied arithmetic, but relatively few studied mathematics beyond the compulsory school leaving age—and of those who did, most were boys. Around 1960, in all countries in the Asia-Pacific region, only a small proportion of students remained at school in the senior secondary classes, and a tradition had been established that only the most intellectually capable senior secondary students would take the non-terminal mathematics subjects. Only the most successful of these senior students subsequently proceeded to a university to study Mathematics, or the Physical Sciences, or in courses such as Engineering, Medicine, Veterinary Science, and Dentistry. School mathematics had become a sieve, through which only the very capable could pass.

But, by the late 1980s, the situation regarding mathematics in the schools of many Western nations had changed dramatically. Retention rates in secondary schools had increased to unprecedented levels, and most senior secondary students chose, or were required, to study some form of mathematics. However, only a small proportion of these chose to continue with higher studies in mathematics or the physical sciences at a university. And many of those who did had often had only moderate achievements in mathematics at school. It seemed that degrees in mathematics or the physical sciences were no longer attractive to many students who had done well in mathematics at school.

However, in 1980 the situation in many developing countries so far as the provision of school mathematics was concerned was very different from that in most Western nations. In the developing nations, many children still did not remain long at primary school. Of the primary school graduates, only a very small proportion continued on to secondary schools and, of these, only a tiny proportion finished up studying senior secondary mathematics. Of those who did, only a tiny élite continued studying mathematics at the tertiary level.

What is most pertinent to this book is that in the 1980s and 1990s almost all the indigenous mathematicians and mathematics education researchers in developing countries were from the tiny élite produced in the way just described. Often, too, politicians and leading education bureaucrats were from the same élite—or, if they were not, they had, at some stage, attended senior secondary schools, at least. Such an élite could not be expected to be adequate representatives of the aspirations, needs and thinking of the vast majority of children who would enter primary and secondary schools in ever-increasing numbers during the last two decades of the twentieth century. To make matters worse, the élite who controlled curricula were prone to visit mathematics "experts" in Western countries, or be visited by them in their own countries. Often these visiting experts came as well-paid consultants.

In most developing nations this curious state of affairs generated abstract

school mathematics curricula which were, arguably, only vaguely connected with the immediate needs of most learners and were often taught by unqualified teachers. The use of pencil-and-paper tests based on these élitist curricula inevitably produced many mathematics education "cripples," for whom the main lesson, learned from years of studying school mathematics, was that they could not do it. At the same time as all this was happening, important school mathematics curriculum issues arose, partly because of the rapid growth in technology and partly because of equity considerations. These were among the factors which contributed to the emergence of the discipline of mathematics education in universities around the world.

An appreciation of this historical background provides the backdrop against which politicians and education bureaucrats have to make policy decisions with respect to mathematics education curriculum issues. But, given the obvious need for more inclusive, mathematics education programs in most nations of the region, what have been the forces operating which have persuaded bureaucrats and governments to retain abstract mathematics curricula, and to assess performance by methods which create legions of "mathematical cripples"?

One of the main driving forces, we believe, has been the common belief that economic prosperity is intimately linked to having rigorous mathematics curricula. Another has been the view, often propagated by professional mathematicians, that standards of numeracy are falling, and that strong government is needed to reverse the situation.

"The Last Bastions of Academic Standards"

Changing Circumstances for Tertiary Departments of Mathematics

Over the past 30 years the levels of mathematical knowledge and skills possessed by students entering mathematics departments in universities in Western nations has tended to decline. In the 1950s it was common for a significant proportion of the highest achieving school graduates to go on to study Pure and/or Applied Mathematics, and Physics, in Science faculties. Gradually, however, there has been a turning away from tertiary mathematics and the natural sciences, towards more vocational programs in areas such as Medicine, Dentistry, Physiotherapy, Veterinary Science, Psychology, Sociology, and—particularly during the 1980s and 1990s—Information Science. As far as we know, this has been a worldwide phenomenon. Many Mathematics Departments now have relatively few graduate students or students who are taking honours degrees, and much of the teaching is done in a "service" capacity—that is to say, to students in other Departments who study mathematics subjects which are vocationally-oriented.

Furthermore, in most nations, and including all nations in the Asia-Pacific region—the number of students completing a senior secondary education has increased year-by-year. The reality is that students entering most universities in the 1990s come from a much broader stratum of society than did those who studied at universities in the 1960s. And, Mathematics Departments no longer get the "pick of the crop." This has created difficulties in tertiary Mathematics Departments, whose

staff are often determined to maintain the standards of yesteryear.

During the 1990s many representatives of Mathematics Departments tended to claim that the students they received from secondary schools were no longer able to cope with the kind of courses that first-year mathematics students had taken in the past. It was claimed that mathematics standards had fallen in the schools. This was often disputed by mathematics educators (see, for example, Ellerton and Clements, 1991b), who pointed out that, because of the higher retention rates in secondary schools and the rise of information science, a whole set of new forces was operating.[25]

Mathematicians—and for that matter some teachers of mathematics—have also claimed that the new teaching approaches and the new methods of assessment used and advocated by some school teachers contribute to the falling standards.

Accusations of Elitism

But although the strident protests from professional mathematicians about the need to respond to these changing circumstances were heard, and often repeated, by politicians, journalists, senior university officials, senior education bureaucrats, and captains of industry, not everyone was prepared to accept what they were saying at face value. In some cases teachers, mathematics educators, senior education bureaucrats, and politicians have been angered by what they have perceived to be élitist tendencies among tertiary mathematicians. Many of these have been prepared to attribute selfish and élitist motives to the opposition of mathematicians to curriculum reform (see, for example, Willis, 1988/1989).

An Australian case study. Willis (1988/1989), an Australian mathematics educator, began an article entitled "Mathematics Should Be More Relevant" by stating that school mathematics had been largely used as a *selection* device, and "we would be hypocritical to pretend otherwise." "But," Willis (1988/1989) added, "surely this is not a defence, rather an indictment of school mathematics" (p. 14).

This set the tone for Willis's article, in which she argued that school mathematics was being used as a selection sieve. She pointed out that although many mathematically strong girls in Australia did not choose to study the more difficult mathematics courses in Year 12, many boys who did take these courses had clearly made an inappropriate choice. Willis went on to argue that the preparatory and selective functions of schooling were "often confused and mathematics appears to be in the centre of this confusion" (p. 14).

In the article Willis criticised the emphasis in primary school arithmetic on drill, claiming that the perceived need by mathematicians to be able to assess objectively was contributing to the maintenance of outmoded curricula and assessment methods. Too much time, she wrote, was being spent on attempting to

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25. It is possible, indeed likely, though, that the smaller proportion of quality students completing majors in Mathematics Departments and then moving on to become secondary teachers of mathematics will affect the quality of teaching and learning in schools. Given that in many nations in the Asia-Pacific region the number of senior secondary students taking mathematics is likely to increase exponentially for at least a decade, the threat of an inadequate supply of knowledgeable, well qualified and skilful senior secondary mathematics teachers, is something that education bureaucrats should take seriously.

turn children "into efficient machines for obtaining answers to mechanical problems," and teachers who tried to do anything different were "roundly criticised for their lack of attention to the basic skills." Yet, Willis (1988/1989) continued, what was really needed was something quite different:

> We need to direct attention to helping children develop sensible methods for calculating but many of the sensible methods may be idiosyncratic (both to themselves and to the task), the majority will be mental methods and most of the rest that combination of mental arithmetic and jottings sometimes called "back of envelope" methods. (p. 15)

In a similar vein, the article questioned the educational validity of the curricula, the teaching methods, and the assessment procedures used in secondary school mathematics.

In a section in her article headed "Mathematics and Privilege" Willis argued that school mathematics serves the interests of a privileged minority—largely middle-class, Caucasian males—and stated that it needed to be changed radically. However, she claimed, attempts to make mathematics more widely accessible were resisted vigorously:

> This tends to take two forms. Those not directly involved in education are likely to ridicule and belittle all such changes, while those who are in education try to block changes and often have the status (and dare I add, the dishonesty) to convince others that they are the last bastions of "academic standards." (p. 16)

Willis (1988/1989) went on to suggest that so long as the major purpose of school mathematics was to sort people, "many of those who are currently privileged by the process will resist attempts to change what happens in schools" (p. 16). She concluded her article by stating that the situation did not make her pessimistic. Willis, who was soon to be a major writer in the development of *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991), predicted: "Mathematics curriculum has and will continue to change for the better. It won't be quick and it won't be uncontested, but it will happen" (p. 16).

What Willis meant in her distinction, in the above display quotation, between those "not directly involved in education" and those who are "in education" was not clear. However, no matter who she meant to include by her use of the term "in education," her use of the word "dishonest" was aggressively consistent with what appeared to be the main thrust of her article—specifically, she seemed to be predicting that change in school mathematics would take place regardless of the tactics which would be used by those who, in Willis' view, currently occupied positions of privilege.

The idea of wresting the control of school curricula (and in particular, school mathematics curricula) from vested interests in universities, was one of the underlying but relatively silent forces in the national curriculum movement in Australia over the period 1989–1996 (see Ellerton & Clements, 1994). Occasionally, the matter was brought into sharp focus. Thus, for example, after a December 1993 meeting of the Australian Education Council (AEC), a spokesperson for a Federal Education Minister was reported as saying that academics needed to realise that they "do not control Years 11 and 12 of secondary school," and "should remember : the majority of Year 12 school leavers will not end up in universities."

According to the spokesperson, "the idea of designing curricula entirely for this purpose [i.e. for preparing students for university study] borders on the offensive."[26]

The proposition that Australian universities had had a stranglehold over school curricula for too long was the subject of an article, which appeared in the *Australian* on March 9, 1994.[27] The article reported that the Chairperson of the Canberra-based Schools Council, Ms Ann Morrow, had stated that senior secondary school students' subject choices were dominated by the prerequisites set by the most desired university faculties. According to the report, Ms Morrow asked: "What level of control, if any, should universities have over the school curriculum, and are the universities justified in wanting some sort of quality control over the numbers and types of students they accept?" Ms Morrow claimed that in view of the fact that the Federal Government wanted to have 95% of young people engaged in education and training by the end of the century, this was a question the education community could no longer ignore.

In the same article, representatives of the Universities of Melbourne, New South Wales and Western Australia were reported as denying that Australian universities controlled school curricula.

The details of the above case study are not important so far as this book is concerned. What is important is how the case study is illustrative of the fact that it can no longer be assumed that decisions on who should control mathematics curricula in schools will be left in the hands of universities or school systems. There are many subtle forces at work in mathematics education, and there can be little doubt that in the Asia-Pacific region mathematics education has always been, and will continue to be, a political matter. There will always be competing forces wanting to control what mathematics is studied in schools, and how it should be assessed.

Mathematics education researchers should do more than merely seek to understand and report the political manoeuvring which shapes policy and practices in mathematics education. They should also seek to have an influence on events. Furthermore, and in line with our advocacy of action research,[28] we believe that mathematics education researchers should publicise pertinent findings of education research, including their own research, and do their best to allow the voices of teachers of mathematics to be heard and attended to. From an equity perspective, mathematics education researchers should work to support moves towards curricula and assessment procedures which are likely to result in more students learning and being empowered by their knowledge and understanding of genuine mathematics. That is not always an easy thing to do, especially when governments control the purse-strings for research awards (Ellerton, 1995a).

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26. Quoted in an article by Eric Aubert titled "States win initiative to develop 'national' school curricula," which appeared in *Campus Review*, December 9–15, 1993, p. 16.

27. Written by Carolyn Jones.

28. See Chapter 5 in this book.

Newman Error Analysis Research, and Implications for the Issue of "What is Basic?"

The achievement of significant change in formal education settings is not easy, and is unlikely to come quickly. It is easy to say that attempts to improve the quality of education should be based on the findings of research, but all too often any implications for practice from education research are not clear. However, in this section we summarise some results of mathematics education research, carried out in a number of Asia-Pacific nations over the past 20 years, which obviously have important implications for mathematics education policy and practice in all nations of the region. The research findings are particularly pertinent to the perplexing issue: "What constitutes basic mathematical skills?"

The research to which we are referring derives from a model developed by Newman (1977a, b), an Australian language educator who, in the mid-1970s, developed a systematic procedure for analysing errors made by students responding to written mathematics tasks. Since 1977 a steady stream of research papers has been published reporting "Newman data" in the Asia-Pacific region: Australia (see, for example, Casey, 1978; Clarkson, 1980; Clements, 1980; Faulkner, 1992; Tuck, 1983; Watson, 1980); Brunei (Mohidin, 1991); India (Clements, 1985; Kaushil, Sajjin Singh & Clements, 1985); Indonesia (Ora, 1992); Malaysia (Ellerton & Clements, 1992b; Kim, 1991; Kownan, 1992; Marinas & Clements, 1990; Sulaiman & Remorin, 1993);[29] Papua New Guinea (Clarkson, 1983, 1991; Clements, 1982); Singapore (Kaur, 1995a); The Philippines (Jiminez, 1992); and Thailand (Singhatat, 1991; Sobhachit, 1991).

The findings of these studies have been sufficiently different from those produced by other error analysis procedures (for example, Hollander, 1978; Lankford, 1974; Radatz, 1979), to attract considerable attention from both the international body of mathematics education researchers (see, for example, Confrey, 1990; Dickson, Brown & Gibson, 1984; Mellin-Olsen, 1987; Zepp, 1989) and teachers of mathematics. In particular, analyses of data based on the Newman procedure have drawn special attention to (a) the influence of language factors on mathematics learning; (b) the inappropriateness of many "remedial" mathematics programs in schools in which there is an over-emphasis on the revision of standard algorithms; and (c) the importance of developing learning environments in which students learn to "mathematise"—that is to say, to be able to determine which mathematics is appropriate for a practical situation, and which boundary conditions apply.

The Newman Procedure

Suppose someone was attempting to answer the following short-answer question which appeared on a pencil-and-paper test:

29. A number of different Newman studies, all carried out in Malaysia in 1993, are reported in Sulaiman and Remorin (1993).

The marked price of a book was $20. However, for a sale, 20% discount on the marked price was given. What was the sale price?

According to Newman (1977a,b, 1983), a person wishing to obtain a correct solution to word problems like this must ultimately proceed according to the following hierarchy:

1. Read the problem;
2. Comprehend what is read;
3. Carry out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy;
4. Apply the process skills demanded by the selected strategy; and
5. Encode the answer in an acceptable written form.

Newman used the word "hierarchy" because failure at any level of the above sequence would prevent problem solvers from progressing towards a satisfactory solution (unless by chance they arrived at a correct solution by faulty reasoning).

Of course, as Casey (1978) pointed out, problem solvers often return to lower stages of the hierarchy when attempting to solve problems. (For example, in the middle of a complicated calculation someone might decide to reread the question to check whether all relevant information has been taken into account.) However, even if some of the steps are revisited during the problem-solving process, the Newman hierarchy provides a fundamental framework for the sequencing of essential steps.

Clements (1980) illustrated the Newman technique with the diagram shown in Figure 1. According to Clements (1980, p. 4), errors due to the form of the question are essentially different from those in the other categories shown in Figure 1 because the source of difficulty resides fundamentally in the question itself rather than in the interaction between the problem solver and the question. This distinction is represented in Figure 1 by the category labelled "Question Form" being placed beside the five-stage hierarchy. Two other categories, "Carelessness" and "Motivation," have also been shown as separate from the hierarchy because, as indicated, such errors can occur at any stage of the problem-solving process. A Careless error, for example, could be a Reading error, a Comprehension error, and so on. Similarly, someone who had read, comprehended and worked out an appropriate strategy for solving a problem might decline to proceed further in the hierarchy because of a lack of motivation. (For example, a problem-solver might exclaim: "What a trivial problem. I can't be bothered doing it.")

Newman (1983) recommended that the following "questions" or requests be used in interviews which are carried out in order to classify students' errors on written mathematical tasks:

1. Please read the question to me. (Reading)
2. Tell me what the question is asking you to do. (Comprehension)
3. Tell me a method you can use to find an answer to the question. (Transformation)
4. Show me how you worked out the answer to the question. Explain to me what you are doing as you do it. (Process Skills)
5. Now write down your answer to the question. (Encoding)

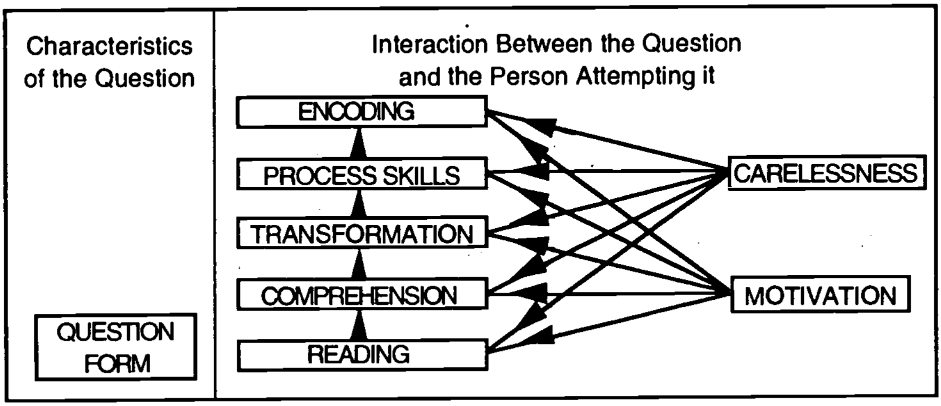

Figure 1: The Newman hierarchy of error causes (from Clements, 1980, p. 4).

If an incorrect response is given to a question then the error is classified according to where the *first* "breakdown" occurred in the attempt to get a solution.[30] If pupils who originally got a question wrong got it right when asked by an interviewer to do it once again, the interviewer should still make the five requests in order to obtain information on whether the original error could be attributed to carelessness or motivational factors.

Example of a Newman Interview

Mellin-Olsen (1987, p. 150) suggested that although the Newman hierarchy was helpful for the teacher, it could conflict with an educator's aspiration "that the learner ought to experience her own capability by developing her own methods and ways." We would maintain that there is no conflict as the Newman hierarchy is not a learning hierarchy in the strict Gagné (1967) sense of that expression. Newman's framework for the analysis of errors was not put forward as a rigid information processing model of problem solving. The framework was meant to complement rather than to challenge descriptions of problem-solving processes such as those offered by Polya (1973). With the Newman approach the researcher is attempting to stand back and observe an individual's problem-solving efforts from a coordinated perspective; Polya (1973) on the other hand, was most interested in elaborating the richness of what Newman called Comprehension and Transformation.

The versatility of the Newman procedure can be seen in the following

30. With the exception of the early study by Casey (1978), in each of the studies cited in this chapter, individual students were interviewed and errors classified according to the *first* break-down point on the Newman hierarchy. With the Casey study, the interviewer helped students over early break-down points to see if they were then able to proceed further towards satisfactory solutions. This was not altogether satisfactory because often the "correction" or strategy suggested by an interviewer was not understood by the student.

interview reported by Ferrer (1991). The student interviewed was an 11-year-old Malaysian primary school girl who had given the response "All" to the question "My brother and I ate a pizza today. I ate only one-quarter of the pizza, but my brother ate two-thirds. How much of the pizza did we eat?" After the student had read the question correctly to the interviewer, the following dialogue took place. (In the transcript, "I" stands for Interviewer, and "S" for Student.)

I: What is the question asking you to do?

S: Uhmm ... It's asking you how many ... how much of the pizza we ate in total?

I: Alright. How did you work that out?

S: By drawing a pizza out ... and by drawing a quarter of it and then make a two-thirds.

I: What sort of sum is it?

S. A problem sum!

I: Is it adding or subtracting or multiplying or dividing?

S: Adding.

I: Could you show me how you worked it out? You said you did a diagram. Could you show me how you did it and what the diagram was?

S: (Draws the diagram in Figure 2A.) I ate one-quarter of the pizza (draws "a quarter").

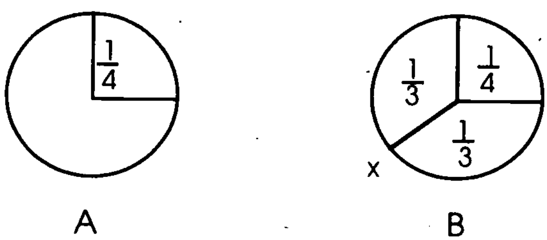

Figure 2: Diagrammatic representations of the pizza problem

I: Which is the quarter?

S: This one. (Points to the appropriate region and labels it 1/4.)

I: How do you know that's a quarter?

S: Because it's one-fourth of the pizza. Then I drew up two-thirds, which my brother ate. (Draws line x - see Figure 2B - and labels each part 1/3)

I: And that's 1/3 and that's 1/3. How do you know it's "one-third"?

S: Because it's a third of a pizza. (From Ferrer, 1991, p. 2)

The interview continued beyond this point, but it was clear from what had been said that the original error should be classified as a Transformation error—the

student comprehended the question, but did not succeed in developing an appropriate strategy. Although the interview was conducted according to the Newman procedure, the interviewer was able to identify some of the student's difficulties without forcing her along a solution path she had not chosen.

Summary of Findings of Early Newman Studies

In her initial study, Newman (1977a) found that Reading, Comprehension, and Transformation errors made by 124 low-achieving Grade 6 pupils accounted for 13%, 22% and 12% respectively of all errors made. Thus, almost half the errors made occurred before the application of process skills. Studies carried out with primary and junior secondary school children by Clements (1980), Watson (1980), and Clarkson (1983) obtained similar results, with about 50% of errors first occurring at the Reading, Comprehension or Transformation stages. Clements's sample included 726 children in Grades 5 to 7 in Melbourne, Watson's study was confined to a preparatory grade in primary school, and Clarkson's sample consisted of 95 Grade 6 students in two Community Schools in Papua New Guinea.

The consistency of the results of these early Newman studies emphasised the robustness of the Newman approach, and drew attention to the importance of language factors in mathematics learning. They also squarely raised the question "What is *really* basic in mathematics education?" For, if about half of the errors made on written mathematical tasks occurred before the application of process skills, then it seemed reasonable to conclude that "remedial" mathematics programs—so widely provided in schools in Asia-Pacific nations—needed to pay more attention to whether the children were able to read and comprehend the language in which the mathematics word problems was couched, and to whether they were able to devise appropriate methods for tackling the problems they were being asked to solve.

Some Recent Newman Data

Marinas and Clements (1990) found that over 70% of initial errors made by a sample of Grade 7 students from Penang in Malaysia were in the Comprehension or Transformation categories, and less than 10% in the Process Skills category. Faulkner (1992), who investigated the errors made by nurses undergoing a calculation audit, in a hospital in Melbourne, obtained a similar result. This was one of the few Newman studies ever reported in which adults were interviewed, and the results, which indicated that the majority of errors made by the nurses were of the Comprehension or Transformation variety, confirmed the earlier conclusion that Process Skills are not nearly as "basic" to survival with dignity in everyday life as many believe they are. A similar conclusion was reached with all the later Newman studies mentioned above (see Ellerton and Clements (1992b) for a summary of the results of some of these studies).

Two Newman studies conducted in 1995 and 1996. The findings of two recent studies, carried out by the writers of this book, in which variations on the Newman procedure were used, warrant attention. For one of the studies (Ellerton & Clements, 1995), 116 Year 8 students, in 12 classes in 5 schools in New South Wales

and Victoria answered a set of written questions. Mathematics teachers in the two schools had agreed that none of the questions should have been too difficult for their students. Half of the questions were in multiple-choice form, and the other half in short-answer form. When Newman interviews were conducted[31] it was found that 80% of errors first occurred at the Reading, Comprehension and Transformation stages, and only 6% of errors first occurred at the Process Skills stage.

This study with Year 8 students was different from previous Newman studies in that Newman interviews were conducted for *all* questions for *all* students in the sample—including those questions for which correct responses had been given. In fact, data obtained from the interviews revealed that for about one-fourth of their correct responses, students either did not understand or had only partial understanding of the concepts and skills which the questions were testing. In such cases, Newman error categories were also attached to these "correct" responses.

One last aspect of this study with Year 8 Australian students is of interest. It was found that different questions produced quite different error patterns. Thus, for example, for the following question, 40% of the errors were of the Process Skills variety, and only 15% were in the Reading or Comprehension or Transformation categories:

> Ice-creams cost 85 cents each, and apples cost 45 cents each. How much altogether would 7 ice-creams and 5 apples cost?

By contrast, for the following question, only 6% of the errors were of the Process Skills variety, but 90% were placed in the Reading or Comprehension or Transformation categories:

> Arrange the following fractions in order of size from smallest to largest:
>
> 1/3 1/4 2/5

Interviews revealed that the Year 8 students replied to fraction questions in a completely instrumental, narrow way. Most simply did not know what the standard m/n symbolism for fractions meant. Many other instances of shallow understanding—or total misunderstanding—were found. For example, interviews for a question involving the calculation of an area revealed that many students could not distinguish between the area and the perimeter of a shape.

The second study (Ellerton & Clements, 1996b) was a comparative investigation of errors made by 101 Year 7 students in two junior secondary schools in Malaysia and 61 students in Year 6 or Year 7 in two primary schools in Perth, Australia. The "same" pencil-and-paper test instrument was used, with the Malaysian version being in *Bahasa Melayu* and the Australian version in English. Every teacher of the pupils agreed that the questions were well worded, and were not too difficult. Newman interviews were conducted by trained interviewers in the language of instruction for the children—*Bahasa Melayu* (in Malaysia) and English (in Perth). Altogether, about 1480 errors were analysed—990 were made by the Malaysian children, and 490 by the Australian children.

31. The interviews were conducted by five trained interviewers. These interviewers provided the Newman error classifications.

The patterns of error classifications in the two samples were strikingly similar, with between 60% and 70% being in either the Comprehension or Transformation categories, and 10% (for the Malaysian sample) and 15% (for the Australian sample) in the Process Skills category.

Additional comments. The variations in these latest studies to the original Newman research methodology—analysing "correct" responses as well as "incorrect" responses, and considering the different error patterns generated by different questions—would appear to have important implications for curriculum and test developers and for teachers. Many "correct" responses are given by students who do not really understand the concepts being tested. Also, teachers, textbook writers and test developers need to become more aware of the kinds of errors students are likely to make on different kinds of tasks.

In all of the Newman studies conducted so far it has been found that error profiles for students obtaining the same total score on a test can be quite different. Thus, for example, two students might get 8 questions correct out of 24 on a test. One of the students could have made 12 Comprehension errors and 4 Transformation errors; the other 12 Process Skills errors and 4 Careless errors. In such a case it is evident that the two students need quite different kinds of remedial help. Such analysis strongly suggests that the common practice of streaming students purely on the basis of "number of questions answered correctly on a standardised test" would be difficult to defend on purely educational grounds.

The high percentage of Comprehension and Transformation errors found in studies using the Newman procedure, in widely differing contexts has, perhaps more than any other body of research, provided unambiguous evidence of the importance of language in the development of mathematical concepts. It appears to be the case, then, that basic skills in mathematics are more to do with receptive and expressive understanding and communication of the language and symbols of mathematics, and less to do with algorithms and calculations, than has been commonly believed. This finding, generated by data obtained in schools in Asia-Pacific nations, could well be one of the most important findings of mathematics education research in recent times.

Newman research has invariably appealed to teachers whose students have been involved because it generates authentic and helpful data. Nevertheless, the findings of the research are not well known among politicians, education policy makers, parents. and even teachers of mathematics. From the Newman studies which have been carried out, thus far, it would appear that the answer to the question: "What are the most basic components of a mathematics education?" is: "learning to read, write, comprehend, and mathematise." In fact, the four operations on natural numbers seem to be well handled by most pupils in upper primary and secondary classes. It should be noted that Newman research has revealed that there is little understanding of, or ability to apply, concepts associated with decimal and vulgar fractions, and percentages.

Findings from Newman research raise the difficult issue of what educators can do to improve learners' comprehension of mathematical text or their ability to transform (that is to say, to identify an appropriate sequence of operations which will solve a given word problem). At present, little progress has been made on this ie, and it should be an important focus of mathematics education researchers in a-Pacific nations over the next decade.

The Death of Proof in School Mathematics?

The previous discussion of Newman categories of errors, and the questions it raised about what is "basic" in school mathematics, paid no attention to the notion of proof, and some mathematicians and educators would assert that of all things, "proof" should be the most basic feature of all school mathematics programs. For, if it is not, then the term "school mathematics" is a misnomer. Most mathematicians believe that proof and proving are at the very heart of mathematics. They also know that there has been a decline in the attention given to proof in school mathematics during the 20th century (Thomas, 1996)

When, early in the 1990s, the Mathematical Sciences Research Institute at the University of California Berkeley held a series of seminars involving mathematicians and high school teachers which was aimed at developing new ways of enticing students into mathematics, one of the most controversial papers was delivered by Lenore Blum, the Institute's deputy director, on the theme: "Are Proofs in High School Geometry Obsolete?" (Horgan, 1993). The mathematicians who attended the Berkeley seminars insisted that proofs were crucial to ensure that a result is true. The high school teachers were less enthusiastic, however, pointing out that students no longer considered traditional, axiomatic proofs to be as convincing as, say, visual arguments. The minutes of the meeting stated: "The high school teachers overwhelmingly declared that most students now (Nintendo/ joystick/MTV generation) do not relate or see the importance of 'proofs'" (Horgan, 1993, p. 82). Horgan (1993) drew attention of his readers to the quotation marks around the final word in this quotation—"proofs."

Throwing Out the Baby with the Bathwater

This discussion thus far in this chapter inevitably raises the question of the role of proof in school mathematics. Might it be the case that proof is such an important aspect of real mathematics that any subject called "Mathematics" which does not emphasise the role of proof is masquerading as something that it is not? The subject might be concerned with practices such as observing, counting, ordering, sorting, and measuring, but *unless* it is concerned with proving, it is not *really* mathematics (Glidden, 1996). In our rush to make mathematics "real" for students, we could be in danger of never introducing the students to real mathematics. Have school mathematics programs lost the baby within the bathwater?

Usiskin (1994b) has argued that, whatever the cultural backgrounds of students, reasoning using deduction should be an essential component of the curriculum for all students, from the first grade up. Students need to grow to understand that proof is needed to decide what is *true* in mathematics (Glidden, 1996; Thompson, 1996). Students also need to learn, through practice, that often there are different ways of proving the same result or theorem, and that one acceptable method can be more "elegant" than another (Tan, 1995).

To avoid dealing with proof in school mathematics would be like teaching science without experiments. As we have argued elsewhere (Clements & Ellerton, 1991), although there is value in asking students to observe patterns and speculate possible rules associated with the patterns, students also need to recognise the

need to generate a deductive argument which establishes a reason why a "rule" must be true. Any school mathematics program in which inductive processes are used exclusively is not *really* teaching mathematics. Students need to gain practice in moving from intuition to proof, as a form of understanding and communication (Schoenfeld, 1994; Thompson, 1996), and a sequence suggested by Mason, Burton and Stacey (1985) is apt: "Convince yourself. Convince a friend. Convince an enemy" (p. 95).

Bertrand Russell (1901) once made a famous remark on the nature of real mathematics:

> Pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is of which it is supposed to be true ... *If* our hypothesis is about anything and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.

Although such an argument would be likely to bring a nod of approval from any mathematician, it does raise difficult questions so far as school mathematics is concerned. Can we expect school children in all parts of the world to be satisfied with a program in which deductive logic plays such an important role? One might also ask if deductive logic is uniformly important in different cultural and linguistic systems, and if it is not then what are the implications of that for mathematics education.

The idea that mathematics is a form of objective knowledge which is only understood through precise and systematic human reasoning has been challenged by curriculum anthropologists such as Bishop (1988) and by philosophers such as Evers and Walker (1983) and Wittgenstein (1969), who have argued that mathematical knowledge is but one aspect of a seamless web of knowledge, and should not be taught as if it were some kind of reified, objective content and relationships. The relativist idea that mathematical knowledge is not absolutely true is at the heart of contemporary constructivist thinking (e.g., von Glasersfeld, 1990). It is also a key aspect of the language-game thesis of Wittgenstein, who, according to Rizvi (1988), maintained that mathematics is merely another language game—one which involves imposing form upon nature, "but the nature itself does not dictate this form: it is we, the human beings, who have a range of capacities and diverse needs, who do" (p. 11).

Curriculum Considerations: Should All Students be Introduced to the Notion of Proof?

The "Working Mathematically" strand of the nationally-developed *Mathematics Profile* for Australia (Curriculum Corporation, 1994) contained the following two outcomes at Levels 7 and 8:

> 7.2 Makes generalisations by abstracting common mathematical features from situations, tests with additional cases and explains why generalisations must be true.

> 8.2 Produces mathematical arguments to convince others of the truth of propositions, including those involving deductions from known information. (p. 4)

Since Levels 7 and 8 were intended for very capable secondary students only, one might ask whether the writers of the *Mathematics Profile* intended that, for most school students in Australia, the notion of "proof" need not (or should not) be given more than passing attention.

Curriculum theorists have tended to argue that school mathematics is a vitally important component of the curriculum for primary and junior secondary students because it attends to the development of content, relationships, and ways of thinking which honour objectivity, consistency, rigour, and the power of deductive reasoning. That is why it has a place in any balanced curriculum (Glidden, 1996).

Phenix (1964), for example, suggested that mathematics expects learners to become involved in "chains of logical reasoning" (p. 73), and that "the symbol systems of mathematics are designed to achieve complete precision in meaning and rigor in reason" (p. 73). According to Phenix (1964), the subject matter of mathematics is "formal abstract symbolic systems within which all possible propositions are consistent with each other" (p. 74). Phenix (1964) added:

> The method of mathematics is essentially postulational. This means that certain postulates, or axioms, are arbitrarily chosen as part of the foundation of a given mathematical system. ... All mathematical reasoning is of the form "if ... then," where the "if" is followed by a postulate (or some necessary inference therefrom) and the "then" is followed by a conclusion, or a theorem. (p. 74)

Phenix went on to say that every mathematical system requires some basis of undefined terms, and this basis, together with the postulates, constitutes what is called the "foundation" of the system. Furthermore, although it is well known that mathematics is of great practical value in science and technology, essentially it is not a "tool" subject, but a discipline in which formal symbolic systems are constructed.

However, it is one thing to talk about the value of students learning to use deductive reasoning, and another to be able to identify what they should be proving. Many experienced teachers of mathematics have found that "proof" is extraordinarily difficult for most students. For example, all over the world, junior secondary school students learn that there are "180 degrees in a triangle"—that is to say, if A, B, and C denote the numbers of degrees in the angles of a plane (not spherical) triangle, then $A + B + C = 180$. If, like Euclid, you think this result can be proved, then consider what you mean by the word "prove." In establishing such a result, what other assumptions would need to be made? Do you think that junior secondary students are ready to engage in the type of thinking involved in proving such a result?

As Bishop (1988) points out, although "180° in a triangle" holds irrespective of geographical context, that does not negate the proposition that the result is culturally-bound. Bishop (1988) also pointedly asks, "Why is it 180° and not 200°?"

The October 1993 issue of *Scientific American* contained an article with the provocative title, "The death of proof" (Horgan, 1993). In the article Ronald L. Graham, of AT & T Laboratories is quoted as saying that there is a trend away from short, clear, conventional proofs in mathematics, and that this trend may be

irreversible. According to Graham, "the things you can prove may be just tiny islands, exceptions, compared to the vast sea of results that cannot be proved by human thought alone" (quoted in Horgan, 1993, p. 82). Horgan maintained that in the future mathematicians seeking to navigate unchartered waters will be likely to depend on experiments, probabilistic proofs and other guides, and it will often be impossible to provide proofs in the classical sense.

The power of the computer has persuaded some mathematicians to predict the death of proof itself (Horgan, 1993). After interviewing several mathematicians, for example, Horgan (1993) claimed that the validity of propositions can now be established by carrying out experiments using computers, and that it is increasingly acceptable for mathematicians to do mathematics without concerning themselves with proof at all. Thus, for example, computers have been used to generate or to validate extraordinarily long proofs, such as Appel and Haken's published proofs of the four-colour theorem and Radziszowski and McKay's solution to the party problem. These proofs required vast amounts of computer time and could not possibly have been developed or even verified by a human being. However, since computers and computer programs are not infallible, it has to be accepted that assertions proved in this way can never be more than provisionally true.

On these matters, and on a host of other matters concerning the role of proof in mathematics and in school mathematics, readers are referred to the excellent article by Gila Hanna and H. Niels Jahnke in the *International Handbook of Mathematics Education* (Hanna & Jahnke, 1996).

Concluding Comments

Well, then, how should we answer the question "What is basic in school mathematics?" The preceding discussion suggests that any answer probably depends not only on the meaning that the person answering the question attaches to the word "basic," but also on the vantage point from which the person answers the question. For example, a secondary teacher with a class of 15 year-olds who struggle to calculate how much change should be given correctly in typical buying and selling transactions, or who are unable (or lack the confidence) to calculate 10% of an amount of money, or who are unable to do clockface calculations, is likely to interpret "basic" in terms of the content and skills the 15-year-olds will need in order to survive with dignity when they leave school.

On the other hand, the mathematician, and the curriculum theorist, who see proof as the basic, ultimate and distinctive characteristic of mathematics, are likely to argue that in their mathematics classes *all* school students should learn to prove. They will maintain that not only is the *idea* of proof basic, but also the ability to know *when* proof is needed, and *how* to prove that certain relationships are necessarily true.

Those mathematics education researchers who are aware of the Newman error analysis literature, on the other hand, are likely to interpret the question "What is basic in school mathematics?" in quite a different way. They will draw attention to

the need to assist children to read and write mathematics, to comprehend and express mathematical text, and to be able to choose sequences of numerical or spatial operations that they can employ to solve problems.

Researchers with a primary interest in ethnomathematics are likely to have a totally different viewpoint from any of the others on the meaning of "basic." They will argue that whatever one's interpretation of the word "basic" might be, the answer to "What is basic in school mathematics?" should vary from culture to culture, from school to school, and even from classroom to classroom within a given culture.

Our conclusion is, therefore, that the expression "basic skills," as it has been applied to mathematics education, is emotive, and ambiguous. In the United States of America opponents of NCTM's *Standards* have referred to them as the "new new math" and have, in the same breath, called for a "return to the basics." Price (1996), as NCTM President, has responded to such claims by saying they represent a dangerous development because "you may recall that a return to the basics is what destroyed the last attempt to improve mathematics education and led to a dramatic downturn in test scores" (p. 538).

Our point in mentioning this here is that Price (1996) went on to say that NCTM's *Standards are* concerned with "the basics." However, with the *Standards*, that term represents far more than the four operations on natural numbers: rather, it refers to "the basics of today and tomorrow" (p. 538).

The theme of this chapter has been that the "basics of today and tomorrow" for school mathematics are not the same in all nations in the Asia-Pacific region. The challenge facing the region's mathematics educators is one of identifying commonalities and differences, and developing mathematics education programs which capitalise on these.

3 Historical Trends in Mathematics Education Research: 1950–1980

Overview

After providing some historical perspectives on the rise and fall of the New Math(s), this chapter will summarise and critique emphases in international mathematics education research during the period 1950–1980. Particular attention will be paid to the absence of an adequate research base to support the introduction of the New Math(s) curricula during that period. It will also be argued that mathematics education research which was carried out in many Western nations during the period tended to be either (a) experimental or survey research in which statistical significance testing was applied; or (b) developmental research in which the theories of Bruner or Piaget were central. The effectiveness of these methods will be an issue for consideration.

In Chapter 4, contemporary trends in international mathematics education research (covering the last two decades) will be described. The worldwide movement towards and then away from the pervasive problem-solving and metacognitive psychological thrusts of the early 1980s will be examined and critiqued, as will the current "constructivist" and sociological "situated cognition" thrusts of the 1990s. Comment will also be made in Chapter 4 on the continuing trend away from quantitative, experimental, theory-driven research towards qualitative, theory-generating research and participatory action research.

The Rise and Fall of the "New Math(s)"

The late 1950s heralded the "New Math(s)" era. Exactly where, when and why the New Math(s) movement began is open to debate: some say it began in the United States following the launching of Sputnik in 1957; others say it originated in France as a result of the efforts of the Bourbaki Group; others point to the efforts of Geoffrey Matthews and Bryan Thwaites in the United Kingdom. It will be shown that there is probably a basis for each of these claims.

Certainly there were similarities in the ways the New Math(s) movement

expressed itself in various countries around the world in the 1960s and 1970s. Whatever the origins, there can be no doubt that a New Math(s) reform, characterised by the introduction of elementary and secondary mathematics curricula which were qualitatively different from previous curricula, gathered momentum in Western nations throughout the 1960s, and in many other countries during the 1970s. The same distinguishing features in the New Math(s) curricula introduced in Western nations in the 1960s were present in the curricula introduced in other countries, including Asia-Pacific nations, during the 1960s and 1970s (Sulemani, 1977).

We shall argue that the New Math(s) curricula in elementary schools tended to have different origins from the New Math(s) curricula in secondary schools. At the elementary school level, the major influence came from developmental psychology and from educationists such as Jerome Bruner, Jean Piaget, Zoltan Dienes and Caleb Gattegno; but at the secondary level, the major influence came from professional mathematicians in tertiary mathematics departments.

Secondary School New Math(s) Curricula

Advocates of the "New Math(s)" usually believed that mathematics was a single entity which should not be presented to learners as compartmentalised areas of knowledge such as arithmetic, algebra, geometry, trigonometry, and calculus. In keeping with this notion of mathematics as an integrated whole, overriding coordinating themes and language forms were introduced. In some countries, set language and set theory came to be regarded as being almost synonymous with New Math(s), and the integrating power of concepts such as function and algebraic structure received considerable attention.

At the secondary level, in particular, New Math(s) courses were characterised by a heavy use of symbolism, and an axiomatic approach to the teaching of number. Paradoxically, there was a swing away from Euclidean geometry in its purest form towards "transformation" geometry which, although still based on Euclid, seemed to require different teaching and learning approaches (Clements, Grimison, & Ellerton, 1989; Lim, 1995; Moon, 1986; Pitman, 1989).

Secondary school New Math(s) programs were invariably introduced by centre-to-periphery methods of curriculum development (Ellerton, Clements, & Skehan, 1989; Popkewitz, 1988). Implementation of the new syllabuses in schools was made compulsory, and textbooks guided much of the New Math(s) instruction and learning that took place. The importance of the new symbolism and approaches was emphasised in externally-set examinations. If professional development for practising teachers responsible for teaching the New Math(s) was provided then it tended to emphasise content—it was thought to be of paramount importance that teachers themselves knew their mathematics and mathematical symbols well, in order that these could be transmitted accurately to learners.

The decision to introduce New Math(s) programs was usually made by Education Ministry officials who had been influenced by mathematicians and politicians. The development and implementation of the programs were left largely in the hands of mathematicians and educators. Teachers were rarely involved in the developmental phase, but once the decision had been made to introduce the courses, they were expected to work hard to understand not only the "new"

mathematical topics (such as probability and statistics), but also the new teaching approaches (such as transformation geometry and the axiomatic treatment of number) and the new symbolism (such as set language and function notation).

In fact, the task of learning new content and trying to use new teaching methods simultaneously, usually without any reduction in normal teaching load, proved to be too much for most teachers of mathematics. The new content was taught in old ways, with set symbols and function notation being used to create the false impression that something new was being done. However, a proportion of teachers responded positively to the New Math(s): for possibly the first time in their careers, they were asked to reflect seriously about issues associated with the teaching and learning of mathematics, and they took up the challenge.

A new era, in which mathematics education would be regarded as an important area of investigation in its own right had dawned. In many countries this era gave birth to a new group of education researchers—those with a special interest in mathematics education.[32]

Elementary School New Math(s) Curricula

The effects of the New Math(s) movement, which was inspired by a desire to provide better links between school and tertiary mathematics curricula, could be seen in mathematics curriculum documents for both elementary and secondary schools. Interestingly, at the elementary school level, but not so much at the secondary school level, the New Math(s) movement was also strongly influenced by the theories of a number of education psychologists. From the 1950s, elementary school mathematics curriculum developers took account of the ideas of educationists and psychologists such as Maria Montessori, Jerome Bruner, George Cuisenaire, Zoltan Dienes, Caleb Gattegno, and Jean Piaget

The educators and psychologists mentioned in the previous paragraph emphasised that students should not be asked to study any topic unless they were developmentally "ready" to do so. Some, like Montessori, Gattegno, Cuisenaire and Dienes, went so far as to develop specially structured materials that, in their view, embodied important mathematical concepts and would assist learners to acquire these concepts. In fact, in some countries, elementary school mathematics curricula which were almost entirely based on a particular type of material were developed—with Dienes blocks and Cuisenaire rods, in particular, being widely used. The common denominator of the theories espoused by all of these psychologists and educationists was that young children learn mathematics best through active physical and mental involvement.

Education Colonialism and the Decline of the New Maths

Critics of the New Math(s) movement often alleged that teachers had been persuaded to concentrate more on teaching children the names of the various field laws for numbers (like, for example, the commutative law for addition, and the distributive law), than on fundamental mathematical knowledge, skills and

32. Mathematics education researchers were already to be found in some countries, especially the United States of America—see Kilpatrick (1992).

principles. The spirit of the time was captured in a popular song by Tom Lehrer which alleged that with the New Math(s), students did not know that 3 + 2 = 5 but did know that addition was commutative and that 3 + 2 was equal to 2 + 3. Although such a caricature was unfair, it was nevertheless widely accepted. It was not surprising, therefore, that in the 1970s, the New Math(s) gave way to a "back-to-the-basics" curriculum thrust in school mathematics.

Despite the fact that tertiary mathematicians had usually played a prominent role in the development of New Math(s) curricula for secondary schools, support for the new curricula was by no means universal among tertiary mathematicians. Indeed, early in the 1960s many leading mathematicians in the United States and in other countries made it clear that they did not support the New Math(s) reforms (Clements, 1992), at either the elementary or secondary level.

Nonetheless, in the late 1960s and the 1970s, when many of the secondary school New Math(s) curriculum reforms were being discarded in the United States, New Math(s) courses were still being introduced in other countries around the world—often with the support of local mathematicians who believed that the mathematics taught in their schools needed to be brought into line with contemporary developments in the rest of the world.

It seems that such were the forces of colonialism that when curriculum innovations in "advanced" countries such as the United States and the United Kingdom were introduced, these innovations were mimicked in other countries after a 10- to 15-year lag period. Thus, for example, in the 1970s Malaysia not only adopted English and Scottish mathematics curricula, but also brought in British advisers to train Ministry of Education personnel and teachers educators in how to use textbooks and other materials (Lee, 1982).[33]

There were good reasons for this. Leading educationists from "advanced" countries were often invited to serve as consultants for the World Bank, the Asian Development Bank, UNESCO, the British Council, etc., and they tended to recommend the curricula and approaches of their own countries to education authorities in the nations where they served (Berman, 1992; Carnoy, 1982; Kitchen, 1995). Also, when leading educationists from "developing" countries undertook masters and doctoral programs of study abroad, they were likely, on their return home, to take steps to "modernise" curricula in their own countries.

The Rise and Fall of Behaviourism in Mathematics Education

Behaviourism in Education, and Individualised Learning Schemes

In fact, much of the post-New Math(s) approach retained some of the New Math(s) emphases. Probability and statistics remained, as did an acceptance of the importance of the function concept. Although the transformation approach to

33. In using the term "colonialism" we are not necessarily accusing the "advanced" countries of forcing their education ideas on others—rather, there was a tendency among the education leaders of many of the other countries to accept such innovations virtually without question.

geometry introduced in the New Math(s) was largely dropped, the old Euclidean axiomatic approach was not re-introduced, much to the concern of many mathematicians. Post-New Math(s) courses emphasised the hierarchical development of basic knowledge, skills, concepts, and principles; they also stressed the importance of repeated practice gained by doing large numbers of examples from textbooks or worksheets.

Curriculum theorists justified this back-to-the-basics movement for school mathematics: and the New Math(s) movement by referring to the work of leading psychologists. The concept of the behavioural objective—that is to say, a succinctly stated expected student outcome—was introduced by Tyler in the early 1930s, who distinguished between eight major types of objectives (Kilpatrick, 1993a). Later, Tyler (1949) specified seven types of behaviour, and argued that every academic course should have two aspects—content and behavioural—and that it was useful for course developers to use a two-dimensional matrix, with one of the dimensions being defined by the seven types of behaviour (Kilpatrick, 1993a).

In the 1950s in the United States of America the use of behavioural objectives in curriculum design and evaluation became particularly important, with the writings of Skinner (1953) being called upon. Skinner's theory, that cause and effect behaviour could be manipulated by conditioning, gave rise to what became known as programmed learning. Other education psychologists also had major influences on curriculum theory at the time. Benjamin Bloom's (1956) *Taxonomy of Educational Objectives* came to be regarded as an education classic, and Robert Gagné's (1967) ideas on learning hierarchies became standard fare in tertiary teacher education courses.

In education, Skinner's theory was an application of the philosophy of behaviourism which, according to Kaufman (1979), is essentially "a translation of Galilean and Newtonian mechanics to the explanation of the relationship between mental events and human action in terms of the chaining of independent, encapsulated entities that follow natural law" (p. 78). Kaufman (1979) went on to summarise the behaviourist philosophy as follows:

1. The environment may be unambiguously characterised in terms of stimuli.

2. Behaviour may be unambiguously characterised in terms of responses.

3. A class of stimuli exists which, applied contingently and immediately following a response, increase it or decrease it in some measurable fashion. These stimuli may be treated as reinforcers.

4. Learning may be completely characterised in terms of various possible couplings among stimuli, responses, and reinforcers.

5. Unless there is definite evidence to the contrary, classes of behaviour may be assumed to be learned, manipulable by the environment, extinguishable, and trainable. (pp. 78–79)

Behaviourist educators are committed to the idea of a scientific universe of stimuli and responses in which learning and understanding are regarded as the result of behavioural adaption stimulated by appropriate reinforcements. They require desired responses to be specified in behavioural terms, and demand that programs developed to assist students to achieve the behavioural objectives. Towards that

end, teachers should reinforce suitable student responses at appropriate times, in order that the desired behaviours will become automated.

Because of the perception that the United States of America had an advanced education system, and because the USA was extensively involved at that time in supporting international aid programs which provided education "consultants" to developing countries, it is not surprising that behaviourism in education quickly became a dominant force in education systems across the world. Thus, for example, in a Commonwealth Secretariat booklet entitled *Mathematics Teaching in Schools*, Taiwo (1974), of the University of Lagos in Nigeria, stated:

> In a new development in curriculum, the general and the particular aims and objectives of the development should be set out. The specific objectives which follow from the general aims should be interpreted in terms of behaviour. Once the behavioural changes are known, test situations can be devised to measure the change. Thus the success of the curriculum in terms of the developers' own aims and objectives can be measured. It is then a separate issue, best done independently, to evaluate the intrinsic value of the original intention of the project as a whole. It is becoming usual to attach to any curriculum development team a psychometrician whose role is to ask pertinent questions of his colleagues, to get them to clarify their aims and objectives, to organise tests of pupils' understanding and of changes in behavior, and to make sure that the feedback of information derived from trials is efficient, adequate and acted upon. (p. 37)

Tyler himself was a consultant to education systems in Thailand, China, Tanzania and Israel. Even after he retired, his consultancy services were widely sought in many countries. His book *Basic Principles of Curriculum and Instruction* was translated into over 25 different languages.

The catchcries in education became "programmed learning," "behavioural objectives," and "individualisation." New Math(s) textbooks were quickly discarded in favour of new textbooks in which behavioural objectives were clearly defined. In many cases, a chapter of a school mathematics textbook would begin with a statement like "By the end of this chapter, you will be able to ...," and a set of twenty or more expected behaviours would be listed. By the end of the 1960s, many Asian mathematics educators were emphasising the importance of stating objectives "in terms of what the pupil should be able to do at the end of the lesson" (National Institute for Educational Research, 1969, p. 26), and mathematics textbook writers and curriculum developers were expected to work within a behaviourist framework.

Mastery Learning

Consistent with Gagné's (1967) learning hierarchy theory, behavioural objectives were sequenced by the curriculum developer and textbook writer (though not always by the teacher) according to some perceived, pre-determined logical structure. Systems of programmed and individualised learning imported into Australia in the 1960s and 1970s, for example, were invariably based on behaviourist theories of learning. The notion of individualised learning was quickly incorporated and systematised into the "mastery learning" ideas of J. B. Carroll, Benjamin Bloom, James Block, and F. S. Keller.

With mastery learning, behavioural objectives for each unit had to be stated

unambiguously, and so too did the level of mastery expected for each topic. Criterion-referenced pre- and post-tests (often of the multiple-choice variety), were carefully constructed so that items corresponded to the stated behavioural objectives. Teaching approaches were expected to be designed around the need to assist students to achieve the stated behavioural objectives. Time, and not ability, was taken to be the major education variable, and it was argued that almost any child could achieve mastery of a unit if he or she was given enough time to do so. Students were given pre-tests before they were taught a new topic, and those who reached mastery level on the pre-test proceeded to the next topic immediately. After a teaching treatment, students were given a post-test and if they reached mastery on this test, they then proceeded to the next topic; if they did not demonstrate mastery, they were then given another teaching treatment followed by another post-test, and so on, until they demonstrated mastery (Hamilton, 1976).

Two different interpretations of mastery learning were particularly influential: Bloom's "Learning for Mastery," and Keller's "Personalised System of Instruction." According to Kulik, Kulik and Bangert-Drowns (1990), in both approaches, material to be learned was divided into short units, with students taking formative tests on each unit. Both forms were widely used in the United States. Perhaps the best known application was the "Individually Prescribed Instruction" (IPI) program developed at the Learning Research and Development Center of the University of Pittsburgh, and first put into operation at Oakleaf School, Baldwin, Pennsylvania. According to Lindvall and Cox (1970), IPI pupils followed a curriculum which had seven elements:

1. The instructional objectives were organised into areas, levels and units.
2. A testing program
3. Prescribed writing procedures
4. Instructional materials and devices
5. Teacher-classroom activity
6. Pupil-classroom activity
7. Classroom management procedures

With "Learning for Mastery" courses it was assumed that material would be presented by the teacher, and that students would move through subject material at a uniform, teacher-controlled pace. Lessons in "Personalised System of Instruction" programs, on the other hand, were presented largely through written materials, with students working individually and at their own rates.

According to the rhetoric, the primary intention of mastery learning approaches was not to compare the achievements of different students, but rather to monitor whether individual students had acquired appropriate behaviours at acceptable mastery levels. Each student was to be given the amount and kind of instruction individually needed.

Bloom (1968), flush with enthusiasm for his concept of mastery learning, predicted that, in mastery classes, 90% of students would achieve at the level previously reached by only the top 10%. Subsequently (see Bloom, 1976), in response to suggestions that such claims were unrealistic and that mastery ponents had not faced squarely the issue of students completing prescribed rses within reasonable time, he changed his stance.

Bloom (1976) then argued that students should not have to put in much more time on school tasks to achieve a 90% level of proficiency, because students with weak backgrounds would need more time to reach proficiency only in the initial stages of a course. Their need for extra time, he said, would diminish as they mastered the fundamental aspects of a topic—for later objectives, all students should be able to learn new material at the same efficient rate because they would all have a sound grasp of the fundamentals.

Mastery learning methods were introduced into education systems throughout the Asia-Pacific region in the late 1970s, and were particularly popular in school mathematics programs. Thus, for example, in Malaysia in 1977 the Ministry of Education and the Universiti Sains Malaysia launched Project InSPIRE, ("Integrated Systems of Programmed Instruction"). It was intended that this project would "assist primary pupils to reach predetermined mastery levels of all topics included in the primary school curriculum" (Lee, 1982, p. 35). Outcomes-based, behaviourist approaches were perceived as a way of helping to restore balance after the perceived excesses of the "New Maths."

In line with the emphasis on mastery learning, considerable attention was paid, in professional development programs for practising teachers, to (a) the writing of succinct behavioural objectives, using Bloom's *Taxonomy*; (b) to the development of criterion-referenced multiple-choice tests; (c) to the organisation of classrooms suitable for individualised learning approaches; and (d) to diagnosis and remediation. Huge investments were made, by textbook publishers and ministries of education, in resources, infrastructure, and professional development programs which were aimed at convincing teachers, schools, and school systems of the need to introduce mastery learning programs.

The Retreat from Mastery Learning Approaches

The apparent logic of mastery approaches to education won many staunch converts during the 1960s and 1970s, but it was not long before practice began to draw attention to theoretical and pragmatic weaknesses.

Criticisms offered on pragmatic grounds. Hamilton (1976) identified five pragmatic weaknesses in the mastery learning approaches of the 1960s and 1970s:

1. The structure of an individualised program was seen to be more important than the professional judgment of teachers involved in the program. Thus, for example, in Sweden, teachers using the Swedish Individualised Mathematics Program were discouraged from moving students from one class to another on the grounds that it would interfere with the research design of those evaluating the program. For the same reason, some teachers were not allowed to carry out separate and additional diagnoses of less able students.

2. Too much intention was paid to intended (that is to say, to pre-specified) outcomes, and unintended but important consequences were often ignored in evaluation processes.[34]

34. Hamilton (1976) cited the example of an objectives-based statistics program which succeeded in assisting the students to become competent in low-level statistics but at the same time turned them away from ever wanting to proceed to higher level courses in statistics.

3. Many important objectives, and particularly those relating to the affective domain (for example, "develop an enthusiasm for mathematics") were not easily specified in behavioural terms.

4. Clear procedures were rarely defined for dealing with students who got progressively further and further behind.

5. No clear criteria were specified for comparing the overall effectiveness of objectives-based curricula with other curricula, which dealt with similar content.

Criticisms on theoretical grounds. Of even greater importance than these criticisms were the criticisms put forward on more theoretical grounds. At a fundamental level, Carl Rogers (1969), in the United States, argued that behaviourist approaches to education were flawed because they worked from the false premise that humans were not free to commit themselves to purposes because they were controlled by factors outside of themselves. Rogers maintained that, from an education perspective, humans should be regarded as being subjectively free, able to make personal choices and responsibly account for the shape of their lives—they are in fact their own architects for living.

Rogers's (1969) ideas were picked up in the United Kingdom by the mathematics educator Edith Biggs, whose classic book (co-authored with J. R. MacLean) carried exactly the same title as Rogers' (*Freedom to Learn*). Inspired by the writings of Jean Piaget, and the writings and teaching of another mathematics educator, Zoltan P. Dienes, Biggs and MacLean (1969) spoke of the need to free students, however young or old, to think for themselves; they also spoke of the need to provide opportunities for students "to discover the order, pattern, and relations which are the very essence of mathematics" (p. 3); they called for education programs which utilised "childlike characteristics such as curiosity." They railed against prevailing assumptions that mathematics was a rigidly sequential subject, and criticised the common practice of assigning certain topics or skills to particular grade levels.

Most significantly, Biggs and MacLean (1969) strenuously criticised American mastery learning approaches whereby behavioural objectives defined what children would be expected to learn to do. According to Biggs and MacLean:

> Providing for individual differences and ensuring continuous progress has been the objective of many organizational schemes in Canada and the United States. These have improved the classroom climate to some extent for some students, but they have not gone beyond the one-text-per-pupil-per-grade concept that has shackled our curriculum for years. In fact, the ideas of a program, whether it be a textbook or a detailed prescriptive syllabus or a course of study, runs counter to current educational thought. What is needed is a broad framework or general guide within which teachers working alone or in groups may develop programs suitable for the children they teach. (p. 6)

Biggs' ideas, and those of Dienes, were influential in some parts of the Asia-Pacific region.

Dienes, who was based in Adelaide in the second half of the 1960s, wrote in the preface to his most influential book, *Building Up Mathematics*, that what he had to say would send "cold shivers down the spines" of those who believed that mathematics is based on logic (Dienes, 1964, p. 11). Dienes, who was inspired by

the ideas of Jean Piaget and had worked closely with Jerome Bruner, argued that children should "learn to distil the mathematical essence from *their* experiences, not from *ours*." He insisted that "each child can have the thrill of being a little Prometheus, stealing the fire from the mathematical gods" (p. 11), a view which placed his philosophy of mathematics education at the opposite end of the educational pole to the behaviourists.

Dienes' view was a variant of the most commonly expressed criticism of behaviourism in mathematics education: behavioural objectives tended to focus on those educational objectives which could be easily stated in terms of expected behaviours, and because of this important affective and higher-order cognitive objectives were in danger of being overlooked. The end result could be that a behaviourist approach to education, in general, and to mathematics education in particular, might lose sight of the total development of learners (Nichols, 1972).[35]

Ausubel and Sullivan (1970) questioned whether a logically structured hierarchical analysis of the skills and behaviours involved in a topic in mathematics necessarily provided the most appropriate pathway for teaching that topic. Ausubel and other researchers were able to demonstrate, for example, that the provision of so-called "advance organisers" could enable learners to study a topic productively even though, from a learning hierarchy perspective, they had not acquired some of the specified "essential prerequisite skills."

Kemmis (1979) questioned the logical and psychological basis for defining "essential pre-requisite skills" for a particular concept or skill. He argued that because learners inevitably have different cultural, linguistic, and educational backgrounds, it makes little sense to suppose that any particular set of learning objectives should be appropriate to a large number of students, from different classes, different schools, and more particularly from different states and nations. For Kemmis (1979), the central problem of individualisation was to respond to each student's own purposes and needs. But, in fact, there may be as many (or even more) learning sequences as there are learners. Individualised learning schemes, on the other hand, are typically based on the unreasonable premise that one, or perhaps several, learning sequences are likely to assist all learners to learn what is necessary to be able to master any particular objective.

Skemp (1976), in a classic paper on instrumental and relational learning in mathematics, attacked the very foundations of behaviourism by pointing out that, if A, B, C, and D are four steps which appear in a learning hierarchy, teaching students to go from A to B, and then from B to C, and then from C to D, did not necessarily assist them to acquire a holistic understanding of how A, B, C and D are related. Indeed, there was no guarantee that someone who had learned each individual step could then return from D to A.

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35. Most supporters of behaviourism in mathematics education (for example, Walbesser, 1972) rejected this criticism. They argued that it was absurd to claim that there existed behaviours which could not be seen, heard, tasted, touched or smelled—and even if such behaviours were found to exist, how could it be decided that someone had achieved them and someone else had not? In any case, Walbesser (1972) stated, it was spurious logic to claim that because behavioural objectives could not be formulated for all instructional purposes it followed that no behavioural objectives should be written.

When, during the 1970s, the writings of Vygotsky (1962) gradually became better known to Western educators, they found that the great Soviet linguist/ educator had carried out studies which had generated data which did not support behaviourist approaches to education in general, and to mathematics education in particular. For example, Vygotsky (1962) wrote:

> The different steps in learning arithmetic may be of unequal value for mental development. It often happens that three or four steps in instruction add little to the child's understanding of arithmetic, and then, with the fifth step, something clicks. The turning points at which a general principle becomes clear to the child cannot be set in advance by the curriculum. The child is not taught the decimal system as such; he is taught to write figures, to add and to multiply to solve problems, and out of all this some general concept of the decimal system eventually emerges (pp. 101–102)

Commenting on this excerpt, Zepp (1989) stated that "perhaps we are fooling ourselves if we concentrate only on quantitatively measurable progress at the expense of the global cognitive development of the child" (p. 57)

Perhaps the most devastating attack on the application of behaviourism and mastery learning to mathematics education, came from Stanley Erlwanger (1975), whose study of the strict application of mastery principles in an elementary school in the United States revealed that children who had succeeded in passing mastery tests lacked any real understanding of what they were doing. They had no idea of links between skills, were unable to apply the mathematics they had "mastered" and, worst of all, had developed a totally inadequate mechanical view of the nature of mathematics. Teachers in the program had been unable to cope with the different demands placed on them by 25 or more children who were working individually in the same classroom, and often they had not realised that their students had acquired faulty or at best very limited conceptions of the mathematics they had been studying.

Freudenthal's criticisms. The eminent Dutch mathematician and mathematics educator, Hans Freudenthal, went one step further by questioning the validity of the research carried out by Benjamin Bloom, James Block, and other mastery learning proponents. In a scathing attack, Freudenthal (1979) not only criticised the concept of mastery learning, but also accused the mastery learning researchers of having applied dubious statistical techniques to bolster their results. Freudenthal was not the first leading educator to question the inferential statistical techniques which had dominated much of Western education research reporting; among others to do so was the Soviet psychologist and mathematics educator, Valdim Krutetskii (1976).

Freudenthal (1978) argued strongly that Bloom's (1956) *Taxonomy of Educational Objectives* should not be applied to mathematics education. He deplored the behaviourists' tendency to atomise school mathematics, poking fun at their propensity for cataloguing hundreds of outcomes. He cited numerous examples of the atomisation of school mathematics evident in school mathematics textbooks in the United States and Europe and, after drawing attention to a report by an American mathematics education researcher who had uncritically accepted stery learning ideas, wrote:

> Who is laughing? Well, a few years ago I still was, and many a mathematician would not be able to understand why nowadays it makes me angry. The mind behind this master work is neither a practical joker nor a crank but a prominent educationist, one of the leaders in American education technique, and this product is not due to a whim or a derailment but is the conscious result of a philosophy of mathematical instruction which is served by a highly developed technique, and for this reason is extremely dangerous. The author of this work has an image of mathematics in his mind which every mathematician will detest from the depth of his (sic) heart—and an image of education which I am sorry to say will please many educationists all over the world. (p. 96)

Freudenthal (1978) went on to say that leading American mathematicians had protested about such atomisation. They had pointed to the likely serious long-term damage to children's images of the nature of mathematics, but their pleas for a reconsideration of the basis of curriculum development had not been heeded. Freudenthal explained that the mathematicians' requests had been largely ignored because too much money had already been spent on curriculum development projects and on textbooks based on mastery approaches.

Ormell's arguments against Bloom's Taxonomy. Despite the arguments against the strict application of behaviourism in mathematics education, there remained many mathematics educators who wished to cling to Bloom's *Taxonomy* as an instrument which promised "to raise education out of an abyss of intuition" (Ormell, 1991, p. 41).[36] In 1974 Christopher Ormell, an English mathematics educator, had a critique of Bloom's *Taxonomy* published in the influential journal, *Educational Research* (Ormell, 1974). Arguing from a philosophical perspective, Ormell stated that the *Taxonomy* debased genuine education in the sense that the picture it painted was likely to leave teachers and educators feeling that the learning-teaching process would be successful if learners could demonstrate, through criterion-referenced tests, the achievement of a specified "bundle of behaviours." Ormell was concerned that teachers and students alike would regard a behavioural objective as like "a chunk of ice held above sea level by a trick," rather than as "the visible tip of an iceberg of thoughts, associations, awarenesses of possibility" (Ormell, 1991, p. 55).

The continuing influence of behaviourist and neo-behaviourist theories of education. Interestingly, despite a strong move by mathematics education researchers in most Western countries in the 1980s away from behaviourism, the intuitive appeal of Bloom's *Taxonomy* and of mastery learning ideas has continued to capture the hearts and minds of educationists in other countries (and this is particularly the case in many Asia-Pacific nations). Even in education systems where mathematics educators have discarded behaviourism, Ministry of Education officials and others, who do not know the pertinent mathematics education literatures, still seek to impose behaviourist, or neo-behaviourist, approaches on school mathematics programs.

The rhetoric associated with the recent national curriculum movements in the United Kingdom and in Australia, for example, has replaced expressions like

36. Both authors have observed that even well into the 1990s, Benjamin Bloom's *Taxonomy of Educational Objectives* has continued to exert great influence on many educators in the Asia-Pacific region.

"behavioural objectives" and "criterion-referenced tests" with "competency-based learning," and "outcomes-based-education" (OBE), but the underlying theory is essentially the same. Although it has appeared in various guises, the behaviourist position is essentially that desirable change is most likely to be achieved by those who (a) specify, in advance, their objectives or outcomes; (b) list behaviour indicators which will enable qualified observers to decide whether such objectives or outcomes have been achieved; and (c) plan a sequence of steps which will facilitate the objectives being achieved. With millions of dollars invested by governments in supporting such approaches, the recent revisiting of behaviourism in education, embellished as it is in new terminology, continues to gather momentum (Ellerton & Clements, 1994).

Educational scholars have a long, and largely unfortunate, record of borrowing perspectives from other disciplines. For quite a few decades, behaviourist, competency-based approaches dominated scholars' interpretations of curriculum development efforts in mathematics—and in other areas of the curriculum—even after most psychologists had concluded that "they were based on simplistic theories that misconstrued the complex nature of human knowing" (Apple, 1995, p. 340). At the present time neo-behaviourist ideas have been taken on by quality management "experts," and such has been the acceptance of their ideas by politicians and education bureaucrats that we now find mathematics teachers and mathematics education researchers being pressured to work within this imposed and inappropriate framework.

There are some voices prepared to make known their to opposition to behaviourist and neo-behaviourist approaches to achieving change in mathematics education. Popkewitz (1988), for example, has maintained that however attractive and compelling behaviourism in mathematics education might be made to sound, the approach is naive and inefficient, and will be abandoned as policy makers discover that, with humans, the contexts of education practice cannot be controlled by social and psychological engineering. According to Popkewitz (1988), "our social conditions contain a host of elements that interact in ways that are never fully specified, predetermined, anticipated, or willed" (pp. 245-246).

The Psychology of Mathematics Education Movement

Early Links Between Psychology and Mathematics Education Research

From at least as early as the 1950s, theoretical commentaries on elementary school mathematics pointed to the need to involve learners in activities which would enable them to link the language and ideas of mathematics with physical materials and the personal worlds of the learners. The work of Cuisenaire and Gattegno in the 1950s, and of Dienes and Bruner in the 1960s, is particularly noteworthy in this regard. In the background, justifying such approaches, were the writings of the Genevan developmental psychologist Jean Piaget, and the Soviet psychologist and linguist, Lev Vygotsky.

Although Piaget was fundamentally a psychologist, and not an educator, he write about the implications of his ideas for education (see, for example, Piaget,

1975), and there can be no doubt that not only did Piaget influence Dienes and Bruner, but also his own original writings were often cited as the main justification for introducing activity-based elementary school mathematics courses.

In the 1970s, mathematics educators in the United States as well as in many other countries, began to accept and extol the developmental principles of Piaget, and the associated ideas of Dienes and Bruner. Those responsible for developing elementary school mathematics programs such as the Nuffield Program, in the United Kingdom, made it clear that these were based on readiness ideas directly associated with the Piagetian notion of stages whereby learners could be classified as sensorimotor, pre-operational, concrete operational, or formal operational.

Certain ways of thinking were linked to each Piagetian stage, and marker tasks (such as conservation of number) were defined by Piaget as being capable of distinguishing learners who were about to or who had already moved from one stage to another. Trainee teachers in all parts of the world, including Asia-Pacific nations, were taught the Piagetian stages in their teacher education courses, and were expected to be able to see evidence of the stages as they observed children during their teaching practice rounds. In a similar way, Dienes's (1964) six stages of learning were thought to provide an underlying framework for understanding how children developed mathematics concepts.

Psychological theory, then, came to be acknowledged as important, both for explaining children's mathematical behaviour and for guiding those responsible for developing elementary school mathematics curricula or writing elementary school mathematics textbooks. There can be little doubt that in the 1970s education researchers in Asia-Pacific nations were strongly influenced by Piagetian stage theory (Cheung, 1980; McGrath, 1980). A typical investigation would contrast the stages of development of Western children with non-Western children. Thus, for example, Sintoovongse (1978), a Thai educator, tested 120 North American and Thai 6 to 11-year-olds, and claimed that the data lent support for Piaget's theory of cognitive development, "because the American and Thai children, at different age levels, achieved similar scores on the classification tasks" (p. 9).

Between 1973 and 1978 the Southeast Asian Ministers of Education Organisation's Regional Centre for Education in Science and Mathematics (RECSAM), co-ordinated a "Concept Learning Project" which was based on Piaget's theory of cognitive development. This Project was dedicated to the task of "seeking information about the status and process of cognitive growth of Southeast Asian children" and to "finding out how children in Southeast Asia learn mathematics and science concepts" (McGrath, Tananone, & Jarig, 1978, p. 17).

This trend toward large-scale Piagetian developmental research was evident in all Asia-Pacific nations. In Papua New Guinea, for example, the "Indigenous Mathematics Project" attempted to determine the cognitive levels of children from all over the nation by classifying them according to how they performed on certain standard Piagetian tasks (see Lancy, 1978; Lancy, Souviney & Kada, 1981).

The Establishment of PME

Although the behaviourist theories of Tyler, Skinner, Gagné, Carroll, Bloom and Block were an altogether different thing from the developmental notions of Bruner, Dienes, Gattegno and Piaget, these two facets of education psychology

together tended to dominate planning for mathematics education programs in schools throughout the world, particularly in the Asia-Pacific region. Given that psychological theories were so highly regarded by mathematics educators of the 1960s and early 1970s, it was hardly surprising when, in 1976, Professor Efraim Fischbein, of the University of Tel Aviv, moved to form an international study group which would bring together psychologists and educators carrying out research in the field of mathematics education. The International Group for the Psychology of Mathematics Education (PME) was formed, with the following three-fold mission:

1. To promote international contacts and the exchange of scientific information in the psychology of mathematics education;
2. To promote and stimulate interdisciplinary research in the aforesaid area with the cooperation of psychologists, mathematicians, and mathematics teachers;
3. To further a deeper and better understanding of the psychological aspects of teaching and learning mathematics and the implications thereof.

Since 1976, many countries have hosted annual PME conferences. Usually between 200 and 500 mathematics educators and psychologists attend the annual conferences, which provide an annual forum for the world's leading mathematics education researchers.

Thus, in the 1970s, the emerging discipline of mathematics education seemed to link itself formally with the established science of psychology. The message was clear: not only should mathematics educators draw from the discipline of mathematics, but they should also draw from various literatures in psychology. Psychological research could inform them not only *what* mathematics learners of various ages might reasonably be expected to learn, but also *the learning environments* which would need to be established in order to maximise the extent and quality of learning.

This link with psychology carried at least one important implication for the fledgling group of mathematics education researchers scattered around the world—if psychology had so many insights to offer mathematics education, then perhaps the research methodologies commonly used by psychologists should be adopted by mathematics educators researchers.

The two most common methodologies used by psychologists were (a) experimental designs, in which inferential statistical analyses were employed, and (b) developmental studies in which small samples were intensively studied in largely qualitative ways. One only has to examine papers published in major mathematics education research journals (for example, *Journal for Research in Mathematics Education*, and *Education Studies in Mathematics*) between 1975 and 1985 to be convinced that these two research paradigms, borrowed from psychology, steered the thinking of many mathematics educators on what constituted acceptable research in their domain. The two approaches became standard for masters and doctoral dissertations written in the 1970s and early 1980s in almost every country in the world where mathematics education research was carried out.

Mathematics Education Research Emerges from a Straitjacket

The view that mathematics education researchers should adopt the methodologies used by psychologists was clearly expressed in two highly

influential books which were published around 1980—Begle's (1979) *Critical Variables in Mathematics Education,* and the collection of articles on *Research in Mathematics Education* edited by Shumway (1980). It could be argued that these books straitjacketed, for some years, thinking within the international mathematics education research community.

In the Preface to Begle's (1979) book the following excerpt from an address by Begle at the first International Congress on Mathematical Education, held at Lyon in 1969, is quoted:

> I see little hope for any further substantial improvements in mathematics education until we turn mathematics education into an experimental science, until we abandon our reliance on philosophical discussion based on dubious assumptions, and instead follow a carefully constructed pattern of observation and speculation, the pattern so successfully employed by the physical and natural scientists.
>
> We need to follow the procedures used by our colleagues in physics, chemistry, biology, etc., in order to build up a theory of mathematics education ... We need to start with extensive, careful, empirical observations of mathematics learning. Any regularities noted in these observations will lead to the formulation of hypotheses. These hypotheses can then be checked against further observations, and refined and sharpened, and so on. To slight either the empirical observations or the theory building would be folly. They must be intertwined at all times.

Begle (1979) lamented the "lack of a solid knowledge base" (p. 156) in mathematics education, and called for mathematics education researchers to develop and test theories in their domain. For him, that was the only way ahead.

Critical Variables in Mathematics Education first appeared in 1979, towards the close of a decade in which "scientific" research methodologies had dominated education research in general, and mathematics education research in particular. In fairness, Begle was only one of many education researchers of his time who were clearly committed to research involving experimental designs and statistical analysis. It is easy in the 1990s to recognise that Begle was defending a perspective on mathematics education research which would be fundamentally challenged in the 1980s, but in the late 1970s most other research methodologies, and particularly the interpretive methodologies, were regarded as being likely to generate only "soft" data for mathematics education.

The publication of *Critical Variables in Mathematics Education* jointly by the Mathematical Association of America and the National Council of Teachers of Mathematics (NCTM) served to legitimise, in the minds of many mathematics education researchers, the notion that the "scientific" approach represented the most appropriate paradigm for education research. According to Begle and Gibb (1980), at the end of the 1970s the field of mathematics education resembled the state of agriculture in the United States several generations earlier. Although mathematics educators drew from "general theories" they had "no established general theory to provide a basis for [their] discussions" (p. 9). Begle and Gibb (1980) believed that what was needed was "the construction of edifices of broad theoretical foundations for the teaching and learning of mathematics as well as curricula, content and organization" (p. 9).

Begle's (1979) call for a renewed emphasis on "scientific" approaches in

mathematics education research came at a time when fundamental questions were being asked about whether this type of research was, in fact, appropriate to education settings (Carver, 1978; Freudenthal, 1979). However, such questions were hardly, if at all, raised in Shumway's (1980) edited collection. A careful reading of this influential volume, which was published by NCTM, makes it clear that at the beginning of the 1980s many North American mathematics educators had committed themselves to a path which, they expected, would lead them to grand theories which would enable mathematics education to become a science.

James Wilson and Jeremy Kilpatrick, both former students of Begle, became editors of the *Journal for Research in Mathematics Education* (JRME). During the time of their editorships, many of the articles published in JRME were of the "scientific," type—in which null and research hypotheses were formulated and tested using inferential statistical techniques. As Wilson (1994) has written, "a large segment of the mathematics education community [between 1976 and 1982, when he was Editor of JRME] had an image of JRME as totally reactive, responding to the field with more concern for research rigor than for the significance of research problems" (p. 2).[37] Wilson (1994) added that "to many folks the image of JRME was quite stereotyped by statistical, methods-comparison research reports."

At the conclusion of *Critical Variables in Mathematics Education*, Begle (1979) included a list of goals for mathematics education. He also suggested five areas which, he believed, needed to be accorded highest priority in mathematics education research. These were:

1. The relationship between teacher knowledge of subject matter and student achievement;
2. Drill;
3. Expository teaching of mathematical objects;
4. Acceleration; and
5. Predictive tests.

Such a list reads strangely in the mid-1990s—the five "critical variables" seem to be so narrow in scope.

Begle's list focused research attention on what many would now believe to be educational wastelands. Vitally important areas of concern, such as the influence of cultural and linguistic factors on mathematics learning, were overlooked or accorded low priority in the scheme of things. Perhaps that is why, for example, over the past 15 years there has not been, in our view, a coordinated series of investigations, among mathematics education researchers in the United States, into how language factors impinge on mathematics teaching and learning (see Ellerton and Clements, 1991a, for an extensive review of pertinent literature).

It was not obvious at the time, however, that Begle's (1979) *Critical Variables in Mathematics Education* and Shumway's (1980) edited collection would, with hindsight, be seen as representing the end of an era. Just as in the mid-1970s the international mathematics education community had had enough of the New

37. As editors of JRME, Wilson and Kilpatrick were clearly constrained by the nature of the articles which were submitted to them for consideration, and by the criteria applied by those within the research community who were called upon to blind-review the submissions.

Math(s), in the mid-1980s it had to decided to leave behind the kind of thinking represented by exhortations of Edward ("Ed.") G. Begle, the man who had come to be known as "the father of the New Math." Begle's five variables became markers, testifying to the close of an influential, formative but hardly creative period in the history of mathematics education research.[38]

Undoubtedly, Begle's exhortations, and the early emphasis on statistically analysed data reported in JRME and other journals, reflected the mood and academic culture of the time. Nevertheless, we believe that it is important that in the new millennium mathematics educators cast off shackles of the past, yet at the same time retain established strengths in their modes of operation.

Statistical Significance Testing in Mathematics Education Research: Some Historical Perspectives

It is not an exaggeration to say that in the 1960s and 1970s most of the relatively small number of mathematics education researchers around the world believed that the best (and perhaps the only legitimate) way to carry out research in education was to design studies which employed statistical significance testing (SST) in the analysis of data. Accordingly, education research methodology courses emphasised the importance of good design, and the use of appropriate SST. At that time, the quest to get a statistically significant result became an end in itself, and the quality of a piece of research, and indeed of a researcher was, somehow, thought to be associated with whether the desired significant result was obtained.

It would be difficult to overstate the effects of this dominance of SST and experimental and quasi-experimental research designs (which employed approaches advocated by Campbell and Stanley, 1963) on the culture surrounding

38. Working Group 24 (which was specifically concerned with identifying criteria for mathematics education research), which met at the 8th International Congress on Mathematical Education (ICME-8), held in Seville (Spain) in 1996, provided an opportunity for Kilpatrick to respond to a paper in which we had made similar comments to those in this Chapter regarding the influence of Begle's (1979) book and Shumway's (1980) edited collection. Kilpatrick maintained, on that occasion, that in 1980 it was recognised by the mathematics education research community that the books represented the end, rather than the beginning, of an era, and that neither of the books had nearly as much influence as we have suggested. We do not agree with Kilpatrick on this point.

One of the writers (Clements) attended a seminar at the 4th International Congress on Mathematical Education (ICME-4), held in Berkeley, California in 1980, which had been organised by Kilpatrick. An important aim of the seminar, which consisted of a series of sessions at which invited papers were read by researchers with outstanding international reputations, was to make Begle's priorities for mathematics education research better known. Mathematics educators from many countries attended the seminar,.

The fact that Begle's (1979) book on *Critical Variables* was sponsored by the Mathematical Association of America and by the National Council of Teachers of Mathematics, further attests to the perceived significance of the book at that time. There can be no doubt that in the Asia-Pacific region, Begle's book was, in the first instance, at least, assumed to be authoritative..

mathematics education researchers in the 1970s and early 1980s. As in many areas of education research in which SST was used, one's chances of having a paper accepted for publication in the top mathematics education research journal, the *Journal for Research in Mathematics Education*, were much greater if statistically significant results were obtained. This, of course, resulted in false impressions being transmitted, because studies in which no statistically significant results were obtained were simply not reported.

Another consequence of the dominance of SST was its effect on the type of research questions which were commonly asked. Questions such as "Is Teaching Method A more effective than Teaching Method B, so far as improving performance on a certain area of mathematics?" were common, as were survey questions of the form, "Does Group X have the same mean performance as Group Y on knowledge and skills represented by test Z?" and "Has performance on basic mathematics skills changed over the past *x* years?" If one ventured into correlational studies using simple or multiple regression, or multivariate analysis (which was a brave thing to do in the days of main frame computers), then one could seek answers to questions such as "What are the best predictors of performance in first-year university mathematics?" These were narrow questions, and answers obtained were hardly likely to change the forms and practices of mathematics education around the world.

Also, since the language and concepts of SST are not simple, articles in which SST-based education research was described were not read by many practising teachers. In fact, most of the readers of SST research reports were other education researchers. It was hardly surprising, therefore, that most educators outside the education research community came to believe that education research was carried out only by people closeted in remote places—like university faculties of education, ministry of education planning offices, or other organisations specialising in educational research.

Two consequences of this attitude were (a) education researchers spent more and more time writing papers to themselves, on increasingly complex topics; and (b) teachers came to believe that they did not need to do research (except if they enrolled for a higher degree). Every now and then educational researchers would make patronising noises suggesting that someone should translate their findings into simpler language, so that ordinary teachers could be made aware of the results of education research.

In the 1970s, then, within education faculties, and within the fledgling world of mathematics education, the parameters for legitimate research were reasonably well defined. Occasionally one came across educationists like Carver (1978), who were courageous enough to question the SST status quo (see, for example, Freudenthal, 1979; Krutetskii, 1976), but such was the strength of the established culture that there still remained the belief, among most "respectable" education researchers, that generally speaking the best educational researchers used SST, and that results established by SST had greater claims to "truth" than results established by other, less quantitative research.

There can be no doubt that in the 1970s and early 1980s, the influence of SST on mathematics education research reached into every nation within the Asia-Pacific region. One only needs to examine early issues of RECSAM's *Journal of Science and*

Mathematics Education in Southeast Asia to find evidence of this (see, for example, Ibe, 1981; Sintoovongse, 1978; Soydhurum, 1980; Talisayon, 1983). Lai and Loo's (1992) review of mathematics research in Malaysia between 1970 and 1990 provided details on 37 studies which were regarded, by the reviewers, as sufficiently rigorous to be worthy of inclusion. Of these 37 studies, 24 were survey-correlational studies, seven were correlational studies, four were case studies, and two were described as test development studies. Most of the studies reviewed incorporated SST analyses.

Around 1980 the pre-eminence of SST-based research began to be questioned by mathematics educators. The appearance in 1976 of an English translation of the main work of the Soviet psychologist and educator, Valdim Krutetskii, together with the ever-increasing recognition that Piaget had had far more to say than "ages for stages," had something to do with this. So too did the increasing acceptance of ethnographic research such as that conducted by Stanley Erlwanger (1975). Slowly, but surely, mathematics education researchers began to ask themselves the kinds of questions about SST that Carver (1978) had asked in his *Harvard Educational Review* article.

The flowering of the constructivist movement in mathematics education during the 1980s was accompanied by a rapid move away from SST research towards more ethnographic and anthropological methods. As mass elementary and secondary education got closer and closer to becoming a reality around the world, many (though certainly not all) mathematics educators, faced with the problem of identifying the implications for practice of a "Mathematics for All" philosophy (D'Ambrosio, 1989, 1994; Usiskin 1994a), moved to embrace more holistic forms of mathematics education. In that context, SST-based research appeared to be no longer appropriate.

Finally, in 1993, came Menon's seminal paper titled "Statistical testing should be discontinued in mathematics education research." He identified and discussed what he called "five myths about SST." According to Menon (1993), SST:

1. does not provide a "controversy-free, unified approach to decision making";
2. says nothing about the probability that a null hypothesis is true;
3. is not based on a logic which parallels the logic of mathematical proof by contradiction;
4. does not necessarily generate reliable/replicable and valid results; and
5. is neither a necessary nor a sufficient criterion for the credibility of research

He argued that SST's contribution to educational research in general, and mathematics education research in particular, had not been beneficial, and that "SST should be discontinued as a tool for such research" (p. 4). In particular, in the process of referring to researchers who assume that statistical significance can be used to demonstrate that their results are "reliable," Menon (1993) maintained that

> it is easy to see why so many fall prey to this illusion of replicability: when a statistically significant result is interpreted to mean that the result was *not* due to chance, the next logical step is to assume that such results can be reproduced by a non-chance (and usually human) agency. In other words, the evidence for the research hypothesis (that is, its validity) is supposedly strengthened by the "resultant" low probability of the null hypothesis. (pp. 10–11)

Clearly, Menon believed that SST-based education research is not worthwhile!

In addition to the five myths, Menon (1993) pointed to a number of harmful effects which arise from the over-reliance of researchers on SST. For example, many researchers pay less attention to the design of the study and rely on looking at *p*-values. However, with such an approach research results can, effectively, be manipulated since almost any study can be made to show significant results if a sufficiently large sample is used—even though such a difference might clearly have no educational significance (Hays, 1974).

It would be wrong to give the impression here that Menon's point of view on SST is accepted by all education researchers (Bourke, 1993; McGaw, 1996). The title of Bourke's (1993) invited response to Menon's paper spoke for itself: "Babies, Bathwater, and Straw Persons: A Response to Menon." However, in the context of this book, two points need to be noted: first, the value of SST (using, for example, t-tests, analysis of variance, or the chi-square non-parametric statistic) is now being seriously questioned by many psychologists and education researchers (see, for example, Clements, 1993; Haig, 1996; Schmidt, 1996; Shea, 1996); and second, Menon works at the Nanyang Technological University in Singapore—which suggests that in the 1990s, opposition to the use of SST in mathematics education was now to found in influential institutions in Southeast Asia.

In fact, since the early 1980s there had been a growing determination among leading mathematics educators in some Asia-Pacific regions to develop more indigenous forms of mathematics education. For example, in Malaysia, Lee (1982) looked forward to "toward the late '80s, [when] one will begin to see the emergence of a bona fide Malaysian system of education that caters for not only the aspirations and needs of the nation but also those of the individuals that make up the nation" (p. 40). There seemed to be a distinct possibility that in the not-too-distant future many of the shackles of colonialism, so evident in the education systems of Asia-Pacific nations in the 1970s, would be overcome.[39]

39. The collection of articles edited by Morris (1980) contains several papers dedicated to the theme.

4 Modern Trends in Mathematics Education Research (Since 1980)

Mathematics Education Research Changes Direction

The failure, in the 1970s, of back-to-the-basics mastery approaches to generate quality mathematics teaching and learning environments stirred leading educators to try to do something which *would* improve the situation. What could be done to make school mathematics more relevant to everyday life, while at the same time providing a better preparation for those who wanted to proceed to further mathematical studies?

Towards New Research Paradigms in Mathematics Education

In 1980, in the United States of America, the National Council of Teachers of Mathematics (NCTM) stated categorically, in *An Agenda for Action: Recommendations for School Mathematics of the 1980s*, that the major challenge for those concerned about the quality of school mathematics was to develop curricula and to provide professional development programs for teachers which would make problem solving the main focus of mathematics teaching and learning (National Council of Teachers of Mathematics, 1980). In the United Kingdom, the Cockcroft Report argued along similar lines (Cockcroft, 1982).

Such are the forces of colonialism that *An Agenda For Action* and the Cockcroft Report precipitated a major surge of interest in problem solving in school mathematics around the world. However, it quickly became apparent that it was difficult for mathematics curriculum developers, teachers, and researchers to define exactly what constituted a mathematics problem, or exactly how a problem-solving mathematics curriculum could be designed, implemented and evaluated. Numerous references were made to generic heuristics, as expounded by the mathematician George Polya (1973), but education researchers and teachers found it difficult to decide whether generalised problem-solving skills could be effectively planned for and taught in mathematics classrooms. Furthermore, the richness of the problem-solving processes evident in children's thinking raised doubts about whether the standard inferential statistical, and developmental

psychological research paradigms were of much use in research on problem solving.

The new emphasis on whether learners were capable of monitoring their own thinking about problem solving forced mathematics educators to ask questions about what was taking place in children's minds as they tried to solve problems. When other factors, such as language, the social and cultural milieu of learning, metacognition, and the use of imagery, for example, came to be recognised as important components of any holistic account of problem solving, it was not long before the old research paradigms were seen to be inadequate for carrying out the needed research.

Reaction Against the Problem-solving Movement

One of the major recommendations in *An Agenda for Action* (National Council of Teachers of Mathematics, 1980) was that mathematics education researchers and funding agencies should give priority in the 1980s to investigations into the nature of mathematical problem solving, and into effective ways of developing problem solvers.

In retrospect, one must ask whether the NCTM's total commitment to a problem-solving focus was premature. How could a professional organisation of mathematics teachers in one of the most "advanced" nations of the world justify its decision to have made such a recommendation when, by its own admission, there was still a need to investigate the nature of problem solving, and to find out effective ways to develop problem solvers?

From this perspective it can be seen that NCTM's *Agenda for Action* was, in fact, an agenda for disaster. All the mistakes made with the "New Math(s)" movement were likely to be repeated. In effect, NCTM was saying "Let's move in a big way into this problem-solving approach—and, as we go, let's clarify what we mean by problem solving in school mathematics."

Surely such clarification should have been made, and accepted, by mathematics educators before such far-reaching recommendations for curriculum change were put forward. We believe that mathematics educators have a history of responding to education rhetoric. Programs based on emotive, but ill-defined concepts—such as "problem solving," "understanding," "discovery learning," "basic skills," "child-centred approaches," "mastery," "outcomes-based-education," and "co-operative learning"—are advocated by politicians, education bureaucrats, teacher organizations, and curriculum developers. Teachers are expected to accept top-down assertions that new curricula need to be implemented immediately.

Almost inevitably, most teachers who identified themselves with the "new" problem-solving movement faced major difficulties at the implementation stage. They were told that children "learned to solve problems by solving problems," and that they should teach their students heuristic strategies based on Polya's four steps (understand the problem; devise a plan; carry out the plan; and look back and check)—or on one of the many published variations of these steps. However, many of the curriculum materials generated in the early 1980s consisted of little more than interesting problems which did not link clearly with existing mathematics curricula and examination expectations. In many schools, "mathematics laboratory

sessions" in which students attempted to solve non-standard problems became once-a-week events, and children's views of what constituted mathematical problem solving came to be based on experiences which occurred during these sessions.

Although writers such as Kilpatrick (1981), and Clements (1981) advised against the premature wholesale adoption of problem solving, it was not until the end of the 1980s that the approaches adopted began to be seriously questioned in the North American mathematics education literature. Interestingly, it was an article by an Australian cognitive scientist, John Sweller, which appeared in the *Journal for Research in Mathematics Education* in 1990, which called into question the research basis for developing a problem-solving approach in which generic heuristic strategies were taught (Sweller, 1990).

Shortly after this, Sweller (1991) published an article titled *Some Modern Myths of Cognition and Instruction* in which he elaborated upon "six myths" which, although widely accepted by mathematics educators, were not in fact supported by the cognitive science literature.

> *Myth 1*. The knowledge explosion is such that we cannot possibly teach people all they need to know because human cognitive capacity is so limited. (p. 72)
>
> *Myth 2*: Because the amount of knowledge we are able to assimilate is so restricted (see Myth 1), our educational systems should emphasise not knowledge, but rather, thinking and problem solving. In this way we will not burden our children with knowledge that will become outdated and irrelevant. (p. 73)
>
> *Myth 3*. Most of the problems presented to school children are, in reality, routine exercises. We should teach children how to solve real problems. (p. 75)
>
> *Myth 4*. We should teach heuristics. (p. 75)
>
> *Myth 5*. Practice at solving many conventional problems is an efficient way of gaining problem-solving expertise. (p. 76)
>
> *Myth 6*. Students fail to demonstrate transfer from one problem-solving task to another because they lack general problem-solving skills. (p. 79)

It would be wrong to give the impression that only negative education consequences derived from the problem-solving movement. There is clearly a sense in which the move towards problem-solving approaches in the teaching and learning of mathematics represented a breath of fresh air after the stultifying mastery learning regime. However, the movement provided yet another example of educators blindly responding to rhetoric and not demanding an acceptable research base.

Another aspect of the problem-solving movement is important in the context of this book. At some time during the 1980s, education authorities in virtually every nation around the world moved to incorporate problem-solving emphases in their school mathematics curricula. The attitude seemed to be that if it was regarded as important in the United States of America, and for education authorities in the United Kingdom and other Western nations, then it was good enough for us, too; we must not be seen to be left behind. This was merely another example of educational colonialism in operation. However, it has stimulated much thinking in

the Asia-Pacific region about the purpose of school mathematics, and has generated research in this region into how problem-posing and problem-solving approaches can be best incorporated into mathematics curricula (see, for example, Fong, 1993, 1994b; Foong, 1993, 1994; Tan, 1995; Teh, 1994).

The impetus for new approaches to mathematics education research generated by the problem-solving thrust was one among many forces which led to a redefinition of what constitutes acceptable mathematics education research (Foong, 1995; Soon Spario, Hang, & Ho, 1995). For some researchers, the shift was one which took them away from carefully designed, large-sample studies employing inferential statistical analysis, or from developmental studies derived from Genevan perspectives, to information processing research (see, for example, Greeno, 1989; Halford, 1993). New cognitive perspectives on how mathematical concepts were developed, and on how problem-solving processes operate in various situations and with different mathematical content were developed. There are still many mathematics education researchers who work within the information-processing paradigm (see English and Halford, 1995, for an overview of this work).

In fact, though, it could be argued that although in the early 1990s the International Group for the Psychology of Mathematics Education (PME) remains a vibrant organisation, the last decade has witnessed a major shift, by mathematics education researchers, away from predominantly psychological, experimental or information-processing, cognitive research. The move has been away from theory-driven research and towards more theory-generating approaches, in which social, cultural and linguistic dimensions are emphasised.

We now summarise some of the directions and methodologies which are making an impact on contemporary mathematics education research.

The Relativist Challenge and the Growth of Constructivist Ideas

The Relativist Challenge

The realisation, around 1980, that the back-to-the-basics, skill-oriented, hierarchical approaches to mathematics teaching and learning were producing a generation of school leavers who had very mechanical, barren views of the nature of mathematics, caused leading mathematicians and mathematics educators to reflect on the images of mathematics they wanted mathematics programs to project.

As it happened, around the same time, questions about the nature of mathematical knowledge, which had always been important in philosophy, began to take centre stage in the worlds of mathematics and mathematics education. That this was indeed the case is supported by the following quotation from the Preface to Morris Kline's (1980) aptly titled book *Mathematics: The Loss of Certainty*:

> Many mathematicians would perhaps prefer to limit the disclosure of the present status of mathematics to members of the family. To air these troubles in public may appear to be in bad taste, as bad as airing one's marital difficulties. But intellectually oriented people must be fully aware of the powers of the tools at their disposal. Recognition of the limitations, as well as the capabilities, of reason is far more beneficial than blind trust, which can lead to false ideologies and even to destruction.

Such uncertainties about the nature of their subject had not been evident among mathematicians at the beginning of the twentieth century, when the Platonic idea that mathematical knowledge, concepts and relationships existed "out-there," independent of human thought and culture, and of physical reality were accepted. Intellectual giants such as David Hilbert and Bertrand Russell had succeeded in convincing most mathematicians that the eternal truths of mathematics were reflected in the laws of nature, and that the main research agenda for twentieth century mathematicians and philosophers was to achieve a succinct statement of the logical development of mathematics (see Russell, 1974).

However, this formalist philosophy of mathematics was shown to be unattainable by Kurt Gödel who, in 1934, proved that, even within such a basic structure as first-order predicate calculus, together with axioms sufficient to model fully the natural numbers, there are statements which cannot be proved, even though they were constructed so that they are true (see Rucker (1982) for detailed comments on Gödel's celebrated theorem).

Gödel's revelation led to a deep questioning of the nature of mathematics. Gödel, himself a dedicated Platonist, was quoted as saying that either "mathematics is too big for the human mind, or the human mind is more than a machine." In a similar vein, Herrman Weyl is reputed to have said: "God exists because mathematics is undoubtedly consistent and the devil exists because we cannot prove the consistency" (quoted in Herlihy, 1986, p. 15). Other statements such as "mathematics is the only branch of theology that has a proof that it is a branch of theology" were made (quoted in Herlihy, 1986, p.15).

In 1930, shortly before Gödel announced his results, Hilbert, when giving a lecture on the nature of human reason, had electrified his audience by exclaiming "We must know! We must know!" Absolute knowledge was the goal of the Hilbertian formalist school. But soon after Gödel "dropped an atom bomb on mathematical foundations" (Nickel, 1985), Herrman Weyl was led to say that although "the question of the ultimate meaning of mathematics remains open, we do not know in what direction it will find its final solution, nor even whether a final objective answer can be expected at all" (quoted in Kline, 1979, p. 1207).

Wittgenstein, Popper, and Lakatos

Wittgenstein, a contemporary of Gödel, after commenting that "there is no religious denomination in which the metaphysical expression has been responsible for so much sin as it has in mathematics" (quoted in Shanker, 1987, p. vii), went on to say that he believed the twentieth century would witness a rejection of the logic of Leibniz, Gottlob Frege, and Bertrand Russell. For Wittgenstein, the Austrian/English philosopher, mathematics was not something entirely independent of

reality, and no statement was true a priori. Western mathematics was merely a product of history, of social transmission processes at work, and had been more or less shaped by a "survival of the fittest" evolutionary process. Forms of mathematics which were accepted, developed, and utilised by a powerful group were passed on not only to children in the group but also to other groups. When a new kind of problem arose, an extension of existing mathematical knowledge was called for, in order that a solution might be obtained; this, in its turn, was socially transmitted, and came to be recognised as "mathematical truth" (Del Campo & Clements, 1990).

So, reducing Wittgenstein's thesis to its simplest form, although physical reality and our biological dispositions impose constraints on the conventions we develop and on the knowledge, skills, and principles we include within what we call mathematics, there is no mathematical reality that guarantees the results that we get (Wittgenstein, 1956, p. 190).

Wittgenstein's prediction that, as the twentieth century progressed, mathematics would come to be seen as a relative rather than as an absolute form of knowledge proved to be right. Karl Popper's fallibilist philosophies, which became increasingly popular from the 1940s onwards, suggested that a scientific theory can never be proved, only refuted. These ideas were taken up in mathematics by Imré Lakatos (1976) who argued that mathematics has been informed and historically held together by a formalist view based on the epistemology of logical positivism (Nickson, 1994). In his book *Proofs and Refutations*, Lakatos (1976) described a long history of disputes within mathematics about the properties of polyhedra, and argued that many mathematicians, in defending the view that mathematics is a form of absolute knowledge, have defined and redefined the term "polyhedron" to fit their goals. In this sense, as Wittgenstein argued, mathematics became a language game used to prop up the myth that in some genuine sense it is a priori, standing apart from other relative states of knowledge (Rizvi, 1988; Watson, 1989).

By 1980, then, it became commonplace for philosophers of science and education to view mathematics as not fundamentally different from knowledge in other domains. Indeed, mathematics was no longer seen as consisting of truths which existed outside the realm of human activity, but rather as domain-specific, context-bound, and as procedurally rooted as any other form of knowledge. Such ideas were only rarely considered by mathematics teachers and educators, however, until they were taken up by so-called "radical constructivists."

And so it has come to be that in the 1990s, mathematics educators, and especially those identified with constructivist ideas, tend to label the Platonist notion that mathematical objects somehow exist independently of human experience as a common misconception. Mathematics educators are now calling for mathematics programs which present mathematics as a socially constructed body of knowledge (Ernest, 1991; Lakoff, 1987; Lerman, 1996a), which has accumulated over the years and "can be found in books, in journals, and in the exchanges in the many different communities of mathematicians" (Bergeron & Herscovics, 1990, p. 125). They are also using the philosophical framework provided by Wittgenstein to explain why children, left to themselves, cannot be reasonably expected to develop "mathematical" concepts which parallel those found in mathematics textbooks (Clements & Del Campo, 1990; Watson, 1989).

Not every mathematician or mathematics educator has accepted relativist views of mathematics, however. For example, Wilson (1986), a British educator with large experience of working in many nations around the world, argued that "mathematical truths are objective, eternal, unchanging, independent of the particular people who happen to be teaching or learning them" (p. 94). Wilson added:

> If all written mathematics were to be destroyed and the whole complex edifice had to be developed again *ab initio*, then the same truths would in due course reappear. Notations would of course differ, so much so that in written form it would not be immediately recognizable to anyone familiar with the "old" mathematics ... [but] the facts and the structures would be the same. (pp. 94–95)

The idea that mathematics is an absolute form of knowledge is still widely accepted. Indeed, it is likely that even in the 1990s a majority of the world's professional mathematicians would endorse Wilson's (1986) view that "mathematics, then, has an absolute quality" and "in this it differs from every other subject in the curriculum" (p. 95).

Mathematics as a Pan-Cultural Phenomenon

Bishop's Argument for a Multiplicity of Different Forms of Mathematics

As some of the above commentary suggests, the answer to the question of whether mathematics is culture free depends critically on how "mathematics" is defined. Does the "mathematics" practised by "mathematicians" in universities around the world define, totally, the scope and essence of mathematics? Alan Bishop (1988) has argued persuasively against such a point of view. For him, mathematics is a pan-cultural phenomenon, something which exists in all cultures; and "Western Mathematics" (which he calls Mathematics with a capital M), is a particular variant of mathematics which has been developed through the ages by various societies.

For Bishop (1988), in addition to "Mathematics" (with a capital M) there are many different kinds of mathematics (with a small m). There is "Chinese mathematics, Greek mathematics, Roman mathematics, African mathematics, Islamic mathematics, Indian mathematics and Neolithic mathematics" (Bishop, 1988, p. 56). Furthermore, as Joseph (1992) has argued strongly, much of Western mathematics as we know it today has non-European roots.

Although Mathematics (capital M) is an internationalised discipline, it nevertheless represents a specific line of knowledge development which has been cultivated by certain culture groups until it has reached the particular form that we know today. It is reasonably well defined, for as Stillwell (1988), an Australian mathematician, has pointed out, probably 99 percent of mathematicians are now in agreement over what is a number, what is a function, etc., and recent claims that Fermat's Last Theorem has finally been proved has demonstrated that mathematicians the world over can agree on what does and what does not constitute a legitimate mathematical proof.

There is a tension between the notion of mathematics as a pan-cultural phenomenon and Mathematics as a discipline practised by mathematicians in universities. Elementary school educators should try to ensure that mathematics curricula link with the personal worlds of young children, but the question remains whether this quest for relevance is still as important as the level of education increases. Should mathematics become less culturally bound as the level of education increases? Can we assume that one of the most important aims of mathematics education is to produce students who know Mathematics with a capital M, and if so, what proportion of school leavers should be in this category? Is it the main aim of school mathematics to produce students who know enough small m and capital M mathematics to be able to cope with everyday situations which demand the application of mathematical skills, including the selection and use of appropriate problem-solving strategies?

An Internationalised Mathematics Curriculum

Although some mathematics education researchers have distinguished between capital M and small m mathematics, many people believe that the *discipline* of mathematics is more or less culture-free, and that there is no good reason why the content of mathematics, and the methods by which it is taught, should vary significantly from region to region and from nation to nation. It is also commonly assumed that the ability to acquire scientific and technological skills is dependent on having an adequate grounding in "capital M" Mathematics (see, for example, Briggs, 1987).

A consequence of the belief that mathematics is culture-free is that many educators believe that it is legitimate for some central authority to prescribe the type, the level and the extent of mathematics that students should be asked to learn. Oldham (1989), after analysing mathematics curricula in the 25 countries which participated in the Second International Mathematics Study (SIMS), concluded that in most national systems of education, mathematics curricula are centrally decreed—by, for example, ministries of education—and that even the theoretical freedom to choose within a prescribed curriculum is usually constrained by the overriding importance of national examinations. Oldham (1989) concluded that there is a considerable degree of cross-national commonality and, although there are some distinct patterns of diversity, "there is indeed an international mathematics curriculum" (p. 212).

The view that ultimately the task of defining mathematics curricula is sufficiently difficult and important that it should not be left to individual teachers, or schools, or local education authorities, would appear to have gathered momentum in recent years. National curricula and assessment procedures have been introduced in the United Kingdom (Noss, 1989); the "Standards for School Mathematics" document has won widespread acceptance in the United States of America (Crosswhite, Dossey, & Frye, 1989; National Council of Teachers of Mathematics Commission on Standards for School Mathematics, 1989; Price, 1996); and there have been official moves in Australia, Canada, and New Zealand, supported by politicians and many educators, to define principles and mathematics entitlements for compulsory and post-compulsory education levels (...rton, 1995; Baxter & Brinkworth, 1989).

Another pointer towards the existence of an implicitly defined international mathematics curriculum is the increasingly widespread use of the distance mode of teaching mathematics, both at school and tertiary levels (Briggs, 1987, p. 26; Ellerton & Clements, 1989b). In particular, the almost universal acceptance of the idea that "quality" mathematics materials (textbooks, for example) prepared in one country can be readily used, without major changes, in another (Swift, 1986) suggests that education authorities everywhere believe that mathematics is more or less the same the world over (Ellerton & Clements, 1989b, p. 4).

The Challenge for Mathematics Curriculum Developers

In this chapter it has been our intention to provide a research basis for discussion of curriculum issues for those directly concerned with the teaching and learning of mathematics, especially in the nations of the Asia-Pacific region. If, indeed, mathematics is not a culture-free phenomenon, then one of the challenges for mathematics education researchers in the Asia-Pacific region is to design and carry out research which will make explicit the implications of that perspective for the revision of mathematics curricula in their own countries. Only then will teachers, curriculum developers, textbook writers, and education administrators be in a position to make more informed decisions about the learning environments that they deem to be educationally appropriate for their students.

Bishop (1988) has maintained in his book, *Mathematical Enculturation*, that mathematics curricula should always take full account of local cultural influences, so that the content and methods of teaching will be consistent with both the nature of mathematics and with research findings on how mathematics is learnt. If his argument is correct, then the acceptance of the notion of an international mathematics curriculum, the existence of national curricula and testing bodies in many countries (including most Asia-Pacific nations), and the moves towards more centralised curricula in the United Kingdom, the United States of America, Canada, Australia and New Zealand are not necessarily advancing the cause of mathematics education. Similarly, the trend towards the use of distance programs for mathematics which adopt and/or adapt curriculum materials developed in cultures different from those of the learners, could be regarded as a retrograde development.

The pertinence of Bishop's argument was underlined by a report in the July 1988 edition of *Commonwealth Education News* on the activities of the Commonwealth Association of Science, Technology and Mathematics Educators (CASTME). According to this report:

> CASTME facilitates the exchange of information among science, mathematics and technology (STM) educators, keeping in mind the great diversity of cultures, customs and technologies across Commonwealth countries. It is particularly concerned with the social implications of STM education. These include the relevance of STM curricula to local needs and conditions, and to the impact of technology, industry and agriculture on a local community.

Despite these sentiments, a paper prepared for the first Board meeting of the Commonwealth of Learning pointed to the fact that many developing countries are ticularly short of scientists, technologists and engineers, and argued that there

are distance-teaching materials already in existence that can be used to provide effective education and training in science and engineering (Commonwealth of Learning, 1988, p. 2). Similar arguments can be, and in fact are being, applied in the area of mathematics education. The commentary provided earlier in this chapter suggests that such arguments are shortsighted and are likely to result in the teaching and learning of inappropriate mathematics.

If different cultures do, in fact, have different forms of "small-m" mathematics then it follows logically that mathematics learning ought not to be immune from the influence of culture. Rather, mathematics should be as culturally bound as learning in any other domain (Bishop, 1988; D'Ambrosio, 1985; Harris, 1991; Stigler & Baranes, 1988). These ideas are in line with the writings of contemporary education philosophers such as Evers and Walker (1983), who argue that mathematical knowledge is but one aspect of a seamless web of knowledge, and should not be taught as if it is an "out-there," objective yet mystical a priori form of knowledge.

Yet, despite the large acceptance of these relativist notions of mathematics among the mathematics education research community, many mathematicians, most curriculum developers, and many teachers still regard mathematics as a culture-free phenomenon, something which represents the pinnacle of human reason (Ellerton & Clements, 1989b). For them, mathematics curricula should be hierarchical in nature, and mathematics teachers should therefore stress the need for students to learn basic mathematical facts and skills, and to make correctly sequenced verbal and written statements (Ellerton & Clements, 1991a).

The flowering of what has come to be known as the "constructivist movement," as well as the anthropological emphasis typified by Bishop's research, have led mathematics education researchers not only to take into account pertinent philosophical, sociological, and critical theory literatures, but also to develop new research paradigms on which they are basing their own research. Thus, there is much mathematics education research in the 1980s and 1990s with a philosophical, anthropological, sociological, and epistemological flavour, and this contrasts with the psychological research emphases of the 1970s. It is simply a matter of fact that since 1980 qualitative methods of research have been increasingly used in mathematics education research.

Constructivism and Mathematics Education Research

As Davis (1990) has stated, anyone "who observes mathematics education has to be impressed by the quite sudden eruption of 'constructivism' as a central concern of so many researchers" (p. 114). While constructivism has probably failed to have more than a peripheral impact on the thinking and practice of most mathematics teachers and curriculum developers, there can be no doubt that over the past decade mathematics education researchers have been challenged to reflect on what a relativist, constructivist mathematics education research agenda might look like (Ellerton & Clements, 1992a).

Each of the keynote addresses at the 1987 annual conference of the International Group for the Psychology of Mathematics Education, for example, was dedicated to the theme, and since then a number of books have been published in which the theoretical bases and implications of constructivism for mathematics education have been explored. In 1990, for instance, a collection of 12 papers was published by the National Council of Teachers of Mathematics in the United States (Davis, Maher, & Noddings, 1990), and in 1991 two major edited collections of papers on constructivism (see Steffe, 1991; von Glasersfeld, 1991b) were published. The Special Interest Group on constructivism which met at the Seventh International Congress on Mathematical Education in 1992 generated much lively debate on the pedagogical implications of constructivism, and papers presented to this Special Interest Group have subsequently been published (Malone & Taylor, 1993). As the 1990s unfolded, the interest in constructivism, especially radical constructivism, continued to be high (see, for example, Mansfield, Pateman, and Bednarz, 1996), although in recent times there have been some strongly expressed critiques—see, for example, Lerman, 1996a; Suchting, 1992; Zevenbergen, 1996).

Radical Constructivism

The radical constructivist movement accepts, as one of its central tenets, the relativist position that mathematics is not an "out-there" pre-existing body of knowledge waiting to be discovered, but rather is something which is personally constructed by individuals in an active way, inwardly and idiosyncratically, as they seek to give meaning to socially accepted notions of what can be regarded as "taken-to-be shared mathematical knowledge." As Ernst von Glasersfeld (1990), possibly the best known advocate of the radical constructivist position in mathematics education, has stated:

> ... knowledge is the result of an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed or instilled by diligent perception or linguistic communication. (p. 37)

According to von Glasersfeld (1990), all good teachers know that guidance which they give to students "necessarily remains tentative and cannot ever approach absolute determination" (p. 37). From a constructivist point of view, there is always more than one way of solving a problem, and problem solvers must approach problem situations from different perspectives.

The views held by constructivists on the nature of knowledge are in harmony with the relativist perspectives of many—though certainly not all—twentieth century philosophers and mathematicians. However, a major distinguishing cornerstone of *radical* constructivist theory is its acceptance of Piaget's emphasis on action (that is to say, all behaviour that changes the knower-known relationship) as the basis of all knowledge. According to this view an individual gets to know the real world only through action (Sinclair, 1990; von Glasersfeld, 1992).

Mathematics Learning Environments and Radical Constructivism

For radical constructivists, the crucial issue is not whether mathematics teachers should allow students to construct their own mathematical knowledge,

"for the simple reason that to learn is to actively construct" (Cobb, 1990a). "Rather," Cobb (1990a) says, "the issue concerns the social and physical characteristics of settings in which students can productively construct mathematical knowledge." (For further commentary on constructivism in general, and on radical constructivism in particular, see, for example, Cobb, 1986; Confrey, 1987; Dorfler, 1987; Kilpatrick, 1987; Labinowicz, 1985; von Glasersfeld, 1983.)

Cobb (1990b) called for constructivist mathematics educators to develop a new context—a "mathematico-anthropological context"—which should facilitate coherent discussion on the specifics of learning and teaching mathematics. According to Cobb there is research support for moving to establish mathematics classroom environments which incorporate the following qualities:

1. Learning should be an interactive as well as a constructive activity—that is to say, there should always be ample opportunity for creative discussion, in which each learner has a genuine voice;
2. Presentation and discussion of conflicting points of view should be encouraged;
3. Reconstructions and verbalisation of mathematical ideas and solutions should be commonplace;
4. Students and teachers should learn to distance themselves from ongoing activities in order to understand alternative interpretations or solutions;
5. Both students and their teachers aim to work towards consensus by which mathematical ideas are linked.

Although many teachers of mathematics would accept all five of these points, all too often the rhetoric of mathematics teachers and educators and the realities of what transpires in mathematics classrooms do not bear much resemblance to each other (Desforges, 1989). Nonetheless, radical constructivists are determined to refine and apply their ideas to mathematics classrooms, however difficult and time-consuming that process might prove to be (Steffe, 1990).

Notwithstanding certain tensions between theory and practice (see Ellerton & Clements, 1992a), the effects of radical constructivism on mathematics education research and on school mathematics are being strongly debated by many mathematics education researchers around the world. In the second half of the 1980s there was a wave of research (see, for example, Wood and Yackel, 1990), which—building on earlier theoretical works (for example, Bauersfeld, 1980; Chomsky, 1957; Mehan & Wood, 1975)—sought to identify the roles of teachers of mathematics who wished to adopt radical constructivist approaches. Cobb (1990a), in a paper entitled "Reconstructing elementary school mathematics," summarised research which attempted to assess the effectiveness of the application of radical constructivist ideas to mathematics teaching and learning. He made five main points:

1. To claim that students can discover mathematics on their own is an absurdity.
2. Students do not learn mathematics by internalising it from objects, pictures, or the like. Mathematics does not reside in learning materials, structured or otherwise.
3. The pedagogical wisdom of the traditional pattern of first teaching [illegible]hematical rules and skills, and then providing opportunities to apply these in

real-life situations, is questionable. An alternative approach takes seriously the observation that from a historical perspective, pragmatic informal mathematical problem solving constituted the basis from which formal, codified mathematics evolved.

4. The teacher should not legitimise just any conceptual action that a student might construct to resolve a personal mathematical problem. This is because mathematics is, from an anthropological perspective, a normative conceptual activity (see Shweder, 1983), and learning mathematics can be seen as a process of acculturation into that practice. This is evident from the fact that certain other societies and social groups have developed routine arithmetical practices which differ from those taught in Western schools.

5. Mathematical thought is a process by which we act on conceptual objects which are themselves a product of our prior conceptual actions. From the very beginning of primary schooling, students should participate in and contribute to a communal mathematical practice which has as its focus the existence, nature of, and relationships between mathematical objects. From this perspective, understanding mathematics involves the process of constructing and acting on what might be called "taken-to-be-shared" mathematical objects.

Other writers have drawn up lists such as this summarising characteristics which should apply to radical constructivist teaching and learning in mathematics. Steffe (1990), for example, has elaborated ten principles for mathematics curriculum design which, he claims, are in keeping with the main thrusts of radical constructivism.

Pateman and Johnson (1990) have claimed that it has been "constructivist" teachers of mathematics who have led the recent important movement towards establishing mathematics learning environments which nurture interest and understanding through co-operation and high quality social interaction. Pateman and Johnson maintained, as did Steffe (1990), that the belief that children construct their own mathematics out of their own actions and their reflections on those actions (in social settings) provides a new framework for those responsible for devising mathematics curriculum and school mathematics programs.

According to Pateman and Johnson (1990), three aspects of curriculum need to be considered—content, methodology and assessment—if environments are to be created which foster socio-cognitive conflict and challenge. If constructivist educators are to be consistent with their own philosophy they cannot rigidly prescribe content in advance; the learning environments they create need to be related to children and to context; and the assessment methods they employ should foster growth and cooperation (which is particularly difficult for teachers and students who are used to giving and receiving competitive ratings). Constructivist teachers need to be opportunists, "willing to continue to learn both about mathematics and children in the attempt to develop them as autonomous creators of their own mathematics" (Pateman & Johnson, 1990, p. 351).

Constructivism in the Historical Contexts of Education Reform

Undoubtedly, radical constructivism has had a large influence on thinking about mathematics education. It cannot be denied, for example, that the recent

NCTM-led *Standards* movement in the United States has been driven by mathematics educators committed to a constructivist position (Bossé, 1995). However, it could be argued that in their missionary zeal to save mathematics education from self-destruction (Kilpatrick, 1987; Phillips, 1995), constructivist mathematics educators have failed to take seriously arguments mounted against constructivist perspectives in general, and, radical constructivist viewpoints in particular.

Constructivist positions have been critiqued from a number of points of view: for unnecessarily downplaying the role of language in communication (Suchting, 1992; Ellerton & Clements, 1992a); for paying only scant attention to the role of motives, goals, values, and needs (Lerman, 1996a); for, on the one hand, over-emphasising the role of *knowing*, which, according to radical constructivist dogma, arises out of an individual's own activity through a process of cognitive dissonance; but, on the other hand, for not paying enough attention to *knowledge*, as something which derives from social, community, linguistic and cultural agreements and shared understandings acquired largely by a process of osmosis (Adler, 1996; Cobb, Perlwitz & Underwood, 1996; Lave & Wenger, 1991; Lerman, 1996a; Nelson, 1993; Smith, 1995; Zevenbergen, 1996); and for over-emphasising the sociopolitical processes or consensus, thereby jettisoning the need for substantial rational justification, and failing to recognise that nature exerts considerable constraint over our knowledge-constructing abilities and allows us to detect and eject our errors about it (Phillips, 1995).

These criticisms are not entirely consistent with each other, of course, because, as Phillips (1995) stated, "constructivism has many sects" (p. 5), each of which harbours some distrust of its rivals.

Bossé (1995) has argued strongly that many educators in the United States of America who led the constructivist push in mathematics education evangelically sought to impose their views on teachers. The teachers, however, were denied the opportunity to be equal partners in the evolutionary process of mathematics education reform. For Bossé (1995), the recent attempt to introduce in schools across the United States of America, the constructivist-oriented NCTM *Standards* — a move so liberally supported by the United States government—was essentially a top-down reform in the sense that the teachers were not involved in policy formation.[40]

Bossé is not the only writer to have rejected claims, so frequently made, that teachers have been democratically involved in curriculum reform processes (see, for example, Ellerton & Clements, 1994; Tyack & Cuban, 1995). A common tactic used by governments (or other influential organisations) to develop a policy for reform, and then offer support to teachers and university educators who are willing to trial materials and approaches in line with the policy. Then it is claimed that the project is supported by teachers and leading educators. This is precisely what happened in the "New Math" of the 1960s, and in the mastery learning and problem-solving movements which followed in the 1970s and 1980s.

40. Such claims are denied by NCTM, which asserts that all teachers and other stakeholders for that matter, have had, and continue to have, the opportunity to be involved, in developments with the *Standards* (see Burrill, 1996).

In the 1990s the call has been for reform-oriented teachers to create constructivist classrooms in which learners are empowered to construct rich mathematical meanings. Bossé's (1995) words are prophetic:

> Today teachers are being shown how to implement the *Standards*; they are not, however, being guided to transcend the *Standards*. Hypothetically, in time, all mathematics educators will be fluent in interpreting and implementing the work of the NCTM. In that day, however, the NCTM, or another more forward-looking group of mathematics educators, will be publishing the next visionary/curricular document. Then the teachers will again be struggling to catch up to the reformers. When teachers are truly considered professionals and they become part of the evolutionary process, then the current game of chase will end. (p. 201)

Or, as Warren (1990) commented, once again educational reform undertook the Herculean task of juggling one ball at a time "but those with a sense of history might have predicted that waiting offstage was a similar act with a different ball to toss around" (p. 78).

Educational reform *can* be more democratic than this, however. Teachers as a group constitute a class of creative and experienced educators, and given the opportunity they can become fully involved in grass-roots education reform which will have the most chance of having affecting practice at the classroom level. That this is indeed the case is evidenced by the action research projects which have been planned, implemented and evaluated by teacher groups in hundreds of schools across Malaysia, as part of the Programme of Innovation, Excellence and Research (PIER) (see Ellerton, Kim, Madzniyah, & Norjiah, 1996). More will be said on this matter in Chapter 5 of this book.

Interactionist Theories

Mathematics education researchers inclined towards social constructivist viewpoints have tended to emphasise the socio-cognitive importance of student-to-student and student-to-teacher interactions (Cobb, 1994b). Because of the Piagetian base of much of their thinking, radical constructivists have tended to argue from a developmental, and, some might say, cognitivist perspective in their attempts to interpret classroom and interview data. However, many contemporary mathematics education researchers distinguish between the social and cognitive dimensions of learning and believe that it is the social influences which are intrinsic to the learning of mathematics (Lerman, 1996a; Voigt, 1994; Zevenbergen, 1996).

The desire to develop interactionist theories to explain and interpret mathematics learning processes has led researchers away from the classical "clinical interview" methodology which was widely used in the early 1980s. Interactionists tend not to prejudge the relative value of different types of learning environments. For them, for example, whole-class environments can facilitate rich mathematical learning. Cobb (1990b, p. 208) has identified the following as potentially worthwhile aspects of whole-class discussions in mathematics classrooms:

1. Explaining how an instructional activity completed by a small group was interpreted and solved;

2. Listening and trying to make sense of explanations given by others;
3. Indicating agreement, disagreement, or failure to understand the interpretations and solutions of others;
4. Attempting to justify a solution and questioning alternatives in situations where a conflict between interpretations or solutions has become apparent.

Although most experienced teachers would like to think that these four aspects are already commonplace in whole-class discussions which occur in their own classrooms, mathematics classroom discourse analyses indicate that this is not the case. Nevertheless, studies of interaction patterns in mathematics classrooms in elementary schools in Japan (see, for example, Easley and Easley, 1992) suggest that quality mathematics learning has occurred in many whole-class teaching and learning environments in classrooms where teachers have never heard of the constructivist movement. If this is indeed the case, then it makes little sense to claim that the radical constructivist movement has been primarily responsible for the development of whole-class teaching methods that are likely to generate high quality learning.

We have argued elsewhere (Ellerton & Clements, 1992a), in fact, that research has not made clear whether developments which have produced such lists are a direct outcome of the radical constructivist movement. As Kilpatrick (1987) has pointed out, the principle that teachers should attempt to create classroom environments in which learners regularly engage in mathematically rich, social interactions has been advocated by many who would not regard themselves (and would not be regarded) as "radical constructivist."

Furthermore, although constructivist educators such as Cobb and Bauersfeld (1995) are strongly in favour of the development of richer social interaction patterns in mathematics classrooms, it is not altogether clear whether this has its origins in their early identification with constructivism or has merely arisen because of their desire to improve the quality of mathematics teaching and learning. Voigt (1994) has claimed that an interactionist perspective on teaching and learning mathematics "is epistemologically compatible with the constructivist psychological perspective" (p. 172), in the sense that both constructivists and interactionist are fundamentally interested in how learners construct meaning.

There is much evidence that at the present time only relatively few mathematics classes in Western nations are such that mathematically rich interactions occur on a regular basis. As Nickson (1994) has stated, large-scale surveys of mathematics classrooms in the United States and the United Kingdom have indicated that in those countries there is a heavy reliance on textbooks and on teaching number facts. Rich teacher-pupil exchanges and on-task student-student interactions are not common; both the teachers and their students share a common focus— of enabling the students to learn to answer standard questions correctly.

The extent to which the same is true of mathematics classrooms in the Asia-Pacific region is a moot point. Aziz's (1987) analysis of primary classrooms in Malaysia revealed teacher-dominated interaction patterns, where agendas were set by teachers, and public interactions were started, led, and closed by teachers. Usually, students were passive "recipients." Das (1987) claimed that probably these findings could "be generalised to cover most of Southeast Asia" (pp. vii).

In Australia, Leder (1988), after analysing interaction patterns in Grade 3 and

Grade 6 primary mathematics classrooms, concluded that her findings were "consistent with those of earlier research" (p. 165), and that as had been found in earlier American and British studies, "boys received more teacher attention" (p. 165). Leder's (1988) paper was subjected to an intensive public review (see Gunstone & Leder, 1992), and although some methodological issues were challenged, all of the reviewers clearly believed that the traditional classroom interaction patterns which Leder's paper revealed were characteristic of what took place in primary mathematics lessons in most Australian schools. Clements and Ellerton's (1995) analysis of a junior secondary mathematics classroom in Australia showed the teacher "channelling" students, in much the same way as has been demonstrated by Voigt (1985, 1994), in European schools.

However, studies of the intricacies of classroom dialogue and its role in the construction of mathematical meaning are extending the concept of "mathematical meaning" to embrace both the individual student's construction of knowledge and the social construction of students as people who think, have feelings about, and use mathematics. Mathematics educators have suddenly become aware of the roles of contexts within intersecting cultures, and this represents a significant shift in the direction and balance of mathematics education research (Zevenbergen, Atweh, Kanes, & Cooper, 1996).

Anthropological Investigations into the Cultural Contexts of Mathematics Education

The "Mathematics for All" Movement

With increasing numbers of students attending elementary and secondary schools around the world, the issue of whether the traditional Western Mathematics curriculum (capital M Mathematics), with its tightly sequenced formal approaches to number, measurement, geometry, algebra and trigonometry, is still appropriate for all learners in all cultures in all countries is receiving more and more attention. Writers on ethnomathematics such as D'Ambrosio (1985, 1989) have argued that, in the past, school mathematics has been an élitist affair, especially suited to the preparation of middle-class males for prestigious professions such as engineering, accounting, and the natural sciences. In that sense it has been a value-laden, effective sieve used for the selection of future leaders.

What is needed, D'Ambrosio (1985, 1994) has argued, is a totally new approach whereby different mathematics curricula are developed, always with the specific needs of existing groups and potential learners in mind. From this point of view, mathematics education has been recognised as having a strong political agenda (Mellin-Olsen, 1987).

Critical theorists such as Frankenstein (1989) have rejected the idea that mathematics is a culture-free phenomenon, and in so doing have pointed out that the sanitised, abstract statements in Western mathematics curricula are but one way of interpreting the world among many competing discourses. What is needed, they say, is a "mathematics for all" approach, in which curricula are developed

which make sense to individual learners *now.* What is *not* needed is a curriculum which sacrifices the future needs of the majority in order to concentrate on those of a tiny minority who will proceed to study high-level mathematics in universities—although, of course, these students should have the opportunity to study courses which meet their immediate and future needs. From the perspective of the "mathematics for all" goal, national, centrally prescribed mathematics curricula are not likely to be satisfactory (Damerow, Dunkley, Nebres, & Werry, 1984).

The title of Clements's (1992) book, *Mathematics for the Minority,* captured the same sentiment—specifically, that in the past, Western school mathematics has catered for the needs of a minority. Elsewhere we have argued (Ellerton & Clements, 1989a) that the effect of such an approach has been that in the past the main lesson learned by most school leavers after many years of being forced to study mathematics at school, was that they could not do it.

It is all too easy to assume, though, that the slogan "mathematics for all" is aimed at developing nations where levels of literacy and numeracy are relatively low. Stake et al's (1993, 1994) case studies in Illinois revealed that many children attending schools in cities in the United States of America are not really learning mathematics, despite the fact that they attend mathematics classes for years. There is a likelihood that the same phenomenon is to be found in many nations.

Stake et al's (1993, 1994) reports suggest that governments cannot easily create conditions through legislation and additional funding that will help the "mathematics for all" objective to be achieved in a short time. In fact, as the reports by Stake and his co-workers at the University of Illinois (see Stake et al., 1993, 1994) seem to demonstrate, too much intervention by government can be counterproductive.

The State of Illinois's first goal for its schools as they entered the 1993-1994 school year read: "By the conclusion of the 1993-94 school year, at least 50% of all students—regardless of race, ethnicity, gender or income status—in each attendance center within the District will score at or above the national norm on a standardized test" (quoted in Stake et al., 1994, p. 42). The State Board of Education exerted great pressure on schools to work hard at improving their mean test scores. But that, in itself, was the main problem, for many school principals, teachers and schools found that they could not cope with the demands which had been placed on them.

Although teachers in many schools—though not necessary all—were willing to work harder than ever before in order to prepare students thoroughly for the forthcoming tests, the mean performance of students at some schools did not improve. The bureaucracy's policies were aimed at achieving school improvement, but in fact their main effect was to harass teachers and students. Conforming to policy requirements took up much of the teachers' time. According to Stake et al. (1994):

> State and District plans for reform called for formalization of the largely intuitive business of teaching. Goals had to be stated. Lessons had to be outlined. Learning had to be measured objectively. It all had to be coded, i.e., put in writing. It was presumed that the act of writing it down and the availability of the written text would facilitate action.

> For many professionals in Chicago as elsewhere, the act of writing it down was artificial, called for oversimplification, invited deceit, and was enormously time consuming. The resulting documents were seldom referred to, partly because they fell so short of their own conceptions of teaching but partly because few teachers and principals had developed habits of working by formal rules and documents. (p. 119)

The teachers did not "own" the change process, and the official aim of ""mathematics for all" was compromised by the very process which was put in place to assist teachers and schools to achieve it.

In the last paragraph, we summarised some of Stake et al's (1993, 1994) attempts to place mathematics education research within a framework which takes into account both the cultural and political forces which surround any mathematics education enterprise. The approach to educational research which attempts to paint a holistic picture—as distinct from approaches which separate elements of cultures—has, of course, been borrowed from anthropology. There can be little doubt that in the past decade, anthropological approaches to mathematics education research have been increasingly adopted.

In anthropological research the researcher attempts to identify, describe, and map the complex web which defines cultures and sub-cultures. Clearly, qualitative methods are often more appropriate for some research investigations than quantitative approaches, although when appropriate, the latter can be used. At the macro level, mathematics education researchers such as Harris (1991) and Watson (1989) have studied the mathematical systems which have developed within various cultural groups, and have been interested in how young children acquire knowledge and understanding of these systems. At the micro level, mathematics education researchers have been especially interested in the sub-cultures in which mathematics teaching and learning occurs.

Researching the Discourses of Mathematics Classrooms

An important example of a sub-culture at the micro level is the mathematics classroom in a formal Western education setting, and in the 1980s a large amount of research aimed at disentangling and understanding complex classroom processes was carried out. As the French mathematics educator, Brousseau (1983, 1984), has argued, children not only bring to classrooms their own mathematical knowledge, but also their knowledge of the ways a particular classroom and a particular teacher operate. Brousseau coined the expression "didactical contract" to describe the way in which pupils create the rules of mathematical classrooms and in so doing define the parameters within which teachers operate on a day-to-day basis. Balacheff (1986), another French researcher, has argued that many students in mathematics classrooms are engaged in a game of convincing their peers and their teacher that they are in control of what they are doing, and that for the students this is often more important than their engagement with the mathematics itself.

This anthropological approach to mathematics education research has been particularly strong in Continental Europe. In Germany, Bauersfeld's (1980) article, "Hidden dimensions in the so-called reality of a mathematics classroom," provided direction for much of the research that would be carried out by German

mathematics education researchers in the 1980s. Bauersfeld (1991) and Voigt (1992) are among many German researchers who have been using anthropological methods to research the culture of mathematics classrooms.

Voigt's (1985) important article on "Patterns and routines in classroom interaction" showed, through analyses of the discourse patterns of a range of mathematics classrooms, that even teachers who were consciously attempting to use constructivist methods tended to impose their own ways of thinking about mathematics on their students. Wood (1994) agreed with Voigt that in many mathematics classrooms teachers do tend to ask questions which fit what has been called the "funnel pattern of interaction"—that is to say, a pattern which generates "a narrowing of joint activity to produce a predetermined answer preferred by the teacher" (p. 155). However, in some classrooms, teachers encourage a more focused pattern of interaction, in which their guiding questions act to focus and stimulate the development of the students' skills and understandings.

The use of anthropological methods in research into mathematics classrooms has not, however, been confined to France and Germany. In the United Kingdom, Nickson (1994) and Lerman (1994), and in Australia Mousley and Clements (1990), are among many researchers who have investigated the cultures of mathematics classrooms. In the United States of America, Cobb (1990b) has written extensively about the need to use anthropological research methods to investigate mathematics classroom environments, and with his co-workers has reported a number of detailed studies which reveal the nature and effects of discourses in elementary mathematics classrooms (see, for example, Cobb, Yackel and Wood, 1988, 1992). Also, over the past 15 years many detailed investigations of mathematics classrooms in Eastern Asian nations have been carried out (see for example, Minato, 1995; Shimizu, 1995).

There is evidence, too, of collaborative mathematics classroom research projects, involving researchers from different countries, being mounted (see, for example, Krummheuer and Yackel, 1990). Easley and Easley (1992), who used discourse analysis techniques to compare discourses in elementary mathematics classrooms in the United States and in Japan, conjectured that the main reason why so many Japanese children perform so well in mathematics is the different interaction patterns which characterise the mathematics classrooms of the two countries. As noted earlier (see Chapter 1), researchers from the University of Chicago (see, for example, Stigler and Baranes, 1988), who have used both discourse analysis and statistical analysis in their comparative studies of mathematics classrooms in the United States, China, Japan and Korea, have reached similar conclusions.

Research (for example, Nickson, 1994) has shown that in formal classroom settings, most teachers of mathematics in Western nations decide what their students should learn and then do their best to help them to learn it. In particular, the teachers ask what they deem to be "appropriate" questions which will assist their students to construct or create for themselves the desired learning. The style of language that the teachers employ is complex, abstract, and quite unlike the language of everyday life (Bereiter, 1970). However, possibly as a consequence of the teachers having experienced mathematics classrooms of this kind for up to 20 years, the teachers themselves are often unaware of this difference in language (Christie, 1985).

Both Bereiter (1970) and Christie (1985) drew attention to how teachers in general, and teachers of mathematics in particular, ask questions for which they already know the answer. However, although the questions are not genuine questions, they are often cognitively demanding, and the teachers want genuine answers (Cummins, 1981). Also, the contexts in which the teachers embed their questions are often seen as contrived or imaginary by the *students*, who fail to appreciate connections which the *teachers* see between the subject content and the students' worlds outside the classroom. Furthermore, although the teachers may use their imagination to compose any question they choose, the students are seldom permitted to provide imaginative answers.[41]

In mathematics classrooms in which students from a range of cultural backgrounds are present, or when the teacher is from a different cultural background from the students, breakdowns in communication are especially likely to occur. For example, Christie's (1985) detailed study of Australian Aboriginal children who were being taught by non-Aboriginal teachers revealed that Aboriginal children were likely to react strongly to being told—by words or in writing—they were wrong, because this is interpreted by the students as meaning "You and the your work are no good." According to Christie (1985):

> The teacher presents feedback in the way he or she has learnt to do, and the child interprets it in the way he or she has learnt to do. But neither understands the other. Not only does this communication breakdown make the children and teachers very unhappy, it also prevents the feedback from being utilised in the purposeful learning process. The teacher resorts to ethnocentric interpretations of the children's behaviour, and the children withdraw into the silent classroom ritual. (p. 68)

Christie (1985) added that his research had revealed that many other subtle cultural conflicts, which powerfully influenced learning and teaching, arose in classrooms.

Thus, for example, whereas Western children have been socialised to recognise goals that are not explicitly stated, Aboriginal children tended "to see the appropriate goal of a classroom exercise as the ritualised performance of the task along predetermined lines (preferably copied from the blackboard, a book, or a competent classmate)" (p. 68). Christie (1985) maintained that for many Aboriginal students, creativity, constructive effort, and individual achievement were irrelevant. He argued that the cultural conflict and communication mismatch had two serious consequences:

> Firstly, it leads them [teachers] to misinterpret much of the children's genuine goal-directed behaviour (that is, consistent with the children's interpretation of the goals) as laziness and cheating. Secondly, they fail to conceptualise the problem in terms of communication breakdown and thus they fail to find a satisfactory solution. (p. 68)

It would be naive to imagine that Christie's analysis is relevant only to classrooms in which Aboriginal children are being taught by Western teachers.

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41. This is not universally true, for the value of asking students to respond imaginatively to more open-ended questions has been recognised by many mathematics education researchers—see, for example, Sullivan and Clarke (1991).

Research of this type points to the likelihood that most teachers of mathematics are not conscious of many of the recurring patterns of discourse which occur in their own classes. Yet, these patterns have a great influence on the extent and quality of mathematics learning which takes place. The onus would appear to be on researchers and teachers to gain a greater understanding of such matters, and then to move to create learning environments which are likely to reduce, or eliminate, those adverse effects on mathematics learning which arise because of communication breakdowns associated with cultural difference. Solutions to this major problem will not come easily, and are likely to vary within and between nations and cultures.

There have been many recent studies in the Asia-Pacific region into the ways students are reacting to what they are expected to learn, and do, in school mathematics lessons (see, for example, Foong, 1987, 1995). At present, there are many who are especially concerned with the diagnosis and remediation of children who are not performing well with respect to the formal skills needed to cope with standard questions, or who are not coping with the transition from one level of schooling to another (see, for example, Fong, 1994a,1995). The next step is to build curricula, and to develop teaching and assessment procedures, which genuinely take account of the backgrounds and needs of *all* students. This will have to continue to be balanced against the contribution that school mathematics makes to national development, but it is now recognised that it is morally questionable to accept an approach in which failure of a certain proportion of students is accepted as desirable.

Although the problem of achieving "mathematics for all" is easy to express, given the vast range of cultures, mathematical heritages, and languages represented in the nations of the Asia-Pacific region, it will be extraordinarily difficult to solve the problem in any meaningful way. The task of achieving equity in mathematics education defied solution throughout the twentieth century—possibly because "equality of educational opportunity" proved to be a notoriously difficult concept to unravel. Mathematics education researchers must not leave it to politicians, education bureaucrats, and psychometricians to formulate "the solution" to the problem of equity for the new millennium. They are being called upon to work with other stakeholders to solve this, the most important problem of all facing mathematics educators in this part of the world and, then, to make known, as persuasively as possible, their solutions to those who have the power to change things.

Situated Cognition Research

Situated cognition research, which combines anthropological and statistical research methodologies and, depending on the researcher, has also called upon critical theory, has continually challenged the thinking and broadened the perspectives of mathematics educators in the 1980s and 1990s. Since mathematics is now being seen by many mathematics educators as a socially constructed body of knowledge, it is appropriate that research aimed at identifying mathematics-related activities in the cultures of learners be carried out. Once this has been done, the findings should be incorporated in curriculum design and classroom activities (Brown, Collins, & Duguid, 1989).

Ideally, the Mathematical (capital "M") skills, concepts and relationships that children are asked to learn in schools should be linked not only with any capital M Mathematics they already know, but also with their immediate, personal worlds (Masanja, 1996; Nunes, 1996; Watson, 1987). If curricula and teaching methodologies are such that capital "M" mathematics is presented in highly formalised ways which do not take account of the backgrounds of the learners, then cognitive links are not likely to be established, and what the children write in response to questions on mathematics examinations will represent nothing more than mere rote knowledge and skills. Although this can assist children to pass examinations—and it may not even do that if the examinations require more than rote recall and a mechanical use of skills—it will be of little practical utility.

On this point the recent "situated cognition" research in mathematics education by researchers such as Carraher (1988), Lave (1988), Peard (1994) and Saxe (1988) has suggested that people in all classes and walks of life are capable of performing quite complex mathematical operations provided that the context in which the mathematics is presented links with the learners' personal worlds. Situated cognition research has revealed, for example, how street children in Brazil who are failing school mathematics (Carraher, 1988; Saxe, 1988), are actually capable of performing quite difficult mental calculations quickly and accurately in their normal out-of-school roles as candy sellers in the streets.

Millroy (1992), in a study of the mathematical ideas and thinking of carpenters, made use of the idea of tacit knowledge. According to Millroy "the physical act of designing and building furniture would involve tacit mathematical knowledge" (p. 13). This concept of tacit knowledge implies that most people know more than they can tell. Tacit knowledge manifests itself though human activities, and is not necessarily expressed or expressible in written or spoken form. Claims that "I know how to do this" can be validated by performance, but are not necessarily invalidated by inability to describe, in written or spoken form, how to do something.

Peard (1994) studied the influence of family background on the thinking of Year 11 students in Australia with respect to probability concepts. One of his samples included a group of students who had been raised in families where participation in gambling activities occurred frequently. He contrasted the way these students approached Year 11 probability problems with the methods used by another group of Year 11 students who came from backgrounds where gambling was not practised, and indeed was frowned upon. Peard found that not only were the "gamblers" able to solve more problems, but they were also able to transfer much of the language and strategies of gambling situations to the mathematics classroom. This was found to be the case even when what was being studied did not appear to have any obvious contextual connections with gambling.

This raises the question whether school mathematics should be generated by societal needs and aspirations rather than be an appendix to it. Or to put this in the form of another question, can mathematics curricula be properly constructed by armchair theorists remote from the action? We believe that an important lesson of history is that the answer to this question is "No."

Mathematics Education in the Asia-Pacific Context

Mathematics Curricula and Culture

Although both authors have worked, over many years now, with mathematics teachers and tertiary mathematics educators in various parts of Asia and the Pacific, we are the first to admit that in any particular nation—including our own (Australia)—we do not understand all the forces influencing mathematics education and the directions and roles of mathematics education research. In an attempt to challenge those who read this book, we shall make some generalisations about our understandings of the forces influencing mathematics educators in the Asia-Pacific region, and comment on the extent to which the present situation is consonant with international trends in mathematics education research.

At the outset we recognise the dangers of what we are about to do. It is often unwise for outsiders to comment on what others are doing, and this is especially true of education in this part of the world, where there are deeply entrenched cultural, social, and religious values which impinge on what happens in schools. We recognise too, that the nations of the Asia-Pacific region are all different, that they have their own education traditions, and that comprehensive summaries of mathematics education research in the region already exist (see, for example, Liau Tet Loke, Sim & Marinas, 1990). Even so, we believe it is useful to present the views of outsiders who have some knowledge of the sensitivities of which they need to take account.

Are mathematics curricula in nations of the Asia-Pacific region of the "mathematics for all" variety, or are they more suited to the needs of a minority of students? To be blunt, it seems to us that school mathematics in most Asia-Pacific nations is based on centralised curricula, frequent pencil-and-paper testing of students, and rather formal textbooks. Neo-behaviourist outcomes-based approaches are increasingly being adopted, and Benjamin Bloom's (1956) *Taxonomy of Educational Objectives: Cognitive Domain* continues to exert a powerful influence over curriculum and test developers.[42]

Outside the classroom the locally constructed versions of mathematics and the internationally recognised formal, capital M Mathematics, impinge on the daily activities of individuals; but *inside* the classroom, capital M Mathematics prevails. For example, the system of Hindu-Arabic numerals is used in schools throughout this region, and facilitates a level of commonality across cultures in mathematical knowledge (Stigler & Baranes, 1988, p. 259).

Pencil-and-paper multiple-choice and short answer tests continue to enjoy a certain mystique, despite the fact that from a mathematical point of view they are invalid because practising mathematicians rarely have to choose one correct answer from four or five given possibilities. Not only that, but as our own research has indicated, almost one-third of student responses are such that either (a) a

42. Note, however, that there is some evidence that Bloom's hold over many mathematics educators in the Asia-Pacific region is at last being seriously questioned (see, for example, Fong, 1994b).

student's response is incorrect but the student, nevertheless has some understanding of the concepts and principles being tested; *or* (b) the response is correct but the student does not fully understand (Ellerton & Clements, 1995).[43]

Many studies have been conducted in which classroom discourses in mathematics classrooms in Japan, China and Korea have been compared with discourses in mathematics classrooms in the United States of America (Chuansheng et al., 1993; Easley & Easley, 1983; Easley & Taylor, 1990; Song & Ginsburg, 1987; Stevenson et al., 1986; Stigler & Baranes, 1988; Stevenson & Stigler, 1992). Generally speaking, those conducting these studies have concluded that interaction patterns in Asian mathematics classrooms are of a different nature, and more conducive to the development of understanding, than patterns in Western classrooms.

However, the validity of the claim that the secret of Asian success in school mathematics lies in classroom discourse patterns should not be taken for granted. Sceptics have claimed, for example, that higher performance on pencil-and-paper tests can be attributed to the fact that students in the schools of some East Asian nations (China and Japan, for instance) receive more instruction in mathematics in schools than do students in Western nations. Statistics reported by Xiaoda (1995), for example, indicated that 13-year-olds in China attend school for about 250 days each year, compared with an average of less than 200 days in most Western nations. Furthermore, 13 year-old Chinese students receive about 300 minutes of mathematics lessons per week, compared with about 200 minutes for Western children (Stevenson & Stigler, 1992). Or, as Yamasaki (1995) expressed it, the total mathematics classroom time for junior secondary students in China is 566 hours, and in Japan, 385 hours: those time allocations are significantly more than the corresponding time allocation for Australian schools (which would be less than 300 hours).

Questions have also been asked about the exact nature of the populations sampled in many of the international comparison studies. In studies of the mathematics achievement of students in Chinese schools, for example, not all of the 29 provinces have been involved; furthermore, over half of the 13-year-olds in China do not attend school, and of those who do, children from minority groups are often excluded from research samples for international studies (Xiaoda, 1995). The influence of important social pressures needs to be taken into account, too, especially those emanating from family, and from the effects of the widespread employment of private tutors.

These and other factors have led some writers (for example, Desforges, 1989) to question the quality of the data on which the conclusions concerning the superior performance of Asian students of mathematics are based. Furthermore, data pointing to quite opposite conclusions have been reported. For example, results of studies into the errors that Indian and Malaysian children make on written mathematical tasks have suggested that teachers of mathematics in India and Malaysia tend to overemphasise rote procedures at the expense of teaching for understanding (Clements, 1985; Lim, 1980; Marinas & Clements, 1990).

43. This research will be discussed in greater detail in Chapter 6.

Is there evidence of colonialist thinking in school Mathematics in the Asia-Pacific region? The history of mathematics education in Commonwealth countries suggests that there was always a tendency among educators in the colonies to mimic what was happening in school mathematics in England (Clements, Grimison & Ellerton, 1989), and we suspect that the same tendency is everpresent in the Asia-Pacific region, where curriculum developments in the United States and the United Kingdom are studied, and often more or less reproduced for local settings. Often the rhetoric (of curricula having been developed which take into account local and national needs and sensitivities) does not match the reality of prescribed curricula and classroom practice. There is even a move towards the definition and adoption of internationalised mathematics curricula (Oldham, 1989).

History provides an excellent example of what we talking about: those attempting to explain why the payment-by-results system (Dear, 1975) was introduced into the Australian colonies in the 1860s need not look far beyond the fact that the system was introduced in England just before its introduction in Australia—it should also be observed that the same system was adopted later in the nineteenth century in many British Commonwealth colonies including India, Ceylon, East Africa and Malaya (Watson, 1982). More recently, all Asia-Pacific nations introduced a version of the "New Math(s)" in the late 1960s or 1970s—everyone wanted to be seen to be keeping up with the rest of the world. This was done despite the fact that New Math(s) was meeting with only limited success in those countries where it had been originally introduced (Moon, 1986).

Clearly, a form of colonialism has been shaping the curricula, assessment policies and indeed the perceived *raison d'etre* of school mathematics in many countries around the world, including the nations of the Asia-Pacific region (Bray & Lee, 1996). Such thinking was not confined to countries politically regarded as colonies, or to the nineteenth or early twentieth century: rather, it was evident in the relationship between many developing countries and so-called "advanced" nations, such as the United States of America, the United Kingdom, and the former United Soviet Socialist Republic. It can be argued that despite the best intentions of all concerned, the employment of mathematics education consultants from "advanced" countries, by UNESCO, the World Bank, the Asian Development Bank, and other similar organisations, for the purpose of advising on mathematics curricula in developing countries has inadvertently nurtured, developed and maintained colonialist attitudes and policies in school mathematics. [44]

Considerations such as these led Clements et al. (1989) to deplore the tendency among Australian mathematics educators to mimic developments in school mathematics in England. That this tendency still existed was apparent throughout the 1980s, when the Cockcroft Report and national assessment and curriculum ideas emanating from the United Kingdom had a major impact on the thinking of

44. Both writers have served as consultants for these organisations, and are aware of the tension induced by the expectation that, on the one hand, consultants are "experts"—who are being paid to advise—yet, on the other hand, they want to assist in the "empowerment" of local education authorities and personnel. It is our experience that everyone does their best under the circumstances, but with all consultancies there are, inevitably, subtle forms of colonialism in operation.

educators in all Australian states. Clements et al. (1989) attributed this tendency, in Australia and elsewhere, to what they called the "colonialist" forces operating among those who seek to define the practices and scope of school mathematics in many countries. They defined "colonialism" as "an attitude of mind accepted by both the leaders and representatives of the colonising power and by those who are colonised, that what goes on 'at home' should also take place in the colonies," and added that while this acceptance is sometimes a conscious act, "more often it is unconscious—people behave in a colonialist way simply because that is the way they have learnt to behave" (p. 72).

According to Clements et al. (1989), one of the most important factors which has contributed to the development and maintenance of colonialist thinking in mathematics education has been the largely unthinking acceptance of the idea that mathematics is a culture-free discipline; that is to say, it has been assumed that mathematics is, and should be, the same wherever it is studied. This kind of thinking has been particularly evident among those responsible for writing reports to bodies responsible for maintaining an overview of educational policy in different countries. For example, Briggs (1987), in a report to the Commonwealth Secretariat in London, stated that it is arguable that mathematics might be particularly suitable to Commonwealth cooperation because "there is no practical requirement and the cultural dependence is less than with other subjects" (Briggs, 1987, p. 27).

Similarly, some notes prepared for the Commonwealth Secretariat in 1986 state, in a section entitled "Foreign teaching material and cultural appropriateness," that there is virtually no danger of straight science, mathematics, or technology courses, that have been developed in one country, being culturally offensive in another" (quoted in Ellerton and Clements, 1989b, p. 4). We believe that such a claim flies in the face of recent anthropological and mathematics education research in which the culture-free idea of mathematics is specifically repudiated (see, for example, Joseph, 1992).

Policies by which mathematics curricula are developed and prescribed externally are, we would contend, not supported by the mathematical, philosophical, professional development and historical literatures to which we have drawn attention. If, mathematics is, indeed, socially constructed then students' efforts to construct mathematics, in order to solve real-life problems, should not be straitjacketed by a rigid, externally prescribed curriculum backed up by a tightly administered external examination system. History suggests that such systems have often developed as a result of colonialist thinking, and have produced only a small proportion of adults who recognise and make use of the power of mathematics.

We ask readers of this book to consider whether this is the case in their own countries. If it is, then we would challenge them to consider whether anything can be done to improve the situation.

Should Asia-Pacific nations have national mathematics curricula? Mellin-Olsen's (1987) arguments raise the thorny issue of the extent to which curriculum decision-making processes should be decentralised. And this issue is not confined to Asia-Pacific nations, for as Mellin-Olsen (1987, p. 123) argues, such matters are equally pertinent in the United States of America and in Europe.

This raises a number of issues concerning mathematics education in the Asia-Pacific region. Within any particular nation, what are the advantages and disadvantages of national mathematics curricula which have been developed and prescribed in that nation? Should basically the same mathematics be taught to groups from very different cultural heritages, but living within the same nation? Do the answers to these questions depend on the level of mathematics education being considered—that is to say, are the answers different for elementary and secondary school mathematics education?

Should Asia-Pacific nations have national mathematics teacher education curricula? Related to the issue of whether Asia-Pacific nations should have a national mathematics curriculum is the issue of whether they should have a standardised, teacher education curriculum, and in particular a common mathematics teacher education curriculum. There are now strong moves around the world to standardise initial teacher education programs, and professional development programs (see, for example, King, 1995; National Project on the Quality of Teaching and Learning, 1996). Those supporting these moves tend to believe in the existence of "basic" teaching competencies, but there are dangers that, in Asia-Pacific nations at least, the ultimate expression of such views would be the imposition of programs which do not really "fit" the cultures and needs of teachers and schools, and therefore the students, within particular education systems.

Take for example, the following item taken from the eighth edition of Lefrancois's (1995a) book of test items for teacher education instructors (p. 266):

> A child who produces several different solutions for a single problem would be thinking:
>
> a. concretely — c. divergently
>
> b. convergently — d. intuitively

We looked in vain for the "(c) *and* (d)" answer. We could even imagine a scenario where "all of these" would be correct. The point is, Lefrancois's (1995a,b) influential approach to teacher education is clearly behaviourist and as such may not be in line with many teacher educators' beliefs about the role of the teacher. However, a national teacher education curriculum has to be created by someone, or some group, and there is a real danger that, because of the blinkered views of those who develop it, approaches to teacher education which are slanted towards a particular view of teaching will be made mandatory.

Changing Patterns in Mathematics Education Research

In this chapter we have pointed to the major move by mathematics education researchers in the 1980s away from statistical, developmental, and behaviourist research paradigms that characterised much of mathematics education research carried out in the 1960s and 1970s. Philosophical, anthropological, sociological and

critical research approaches, involving the analysis of qualitative rather than quantitative data, began to be more widely adopted. Increasingly, the major aim of research projects was to gain a holistic understanding of the sub-cultures within which mathematics teaching and learning occurred. It is our belief that these qualitative research paradigms need to be adopted by more Asia-Pacific mathematics education researchers.

Another message carried in this chapter is that the international mathematics education research of the last two decades has made it abundantly clear that mechanistic and prescriptive approaches to mathematics curriculum development and assessment, often accompanied by tightly controlled national curriculum and assessment practices, and employing behaviourist language (such as outcomes and competency-based learning), do not work. They fail because when they are in place too many teachers teach for the lowest common denominator. The teachers note the skills prescribed in the curriculum, focus on those skills that they know will be tested, and teach those to all students in their classrooms, irrespective of whether the mathematics being presented is culturally or cognitively appropriate for the learners.

Such an approach inevitably produces a majority of school leavers who feel that they cannot do mathematics, and fosters an attitude that mathematics is an "out-there," reified body of knowledge which is useful only for very intelligent people in certain professions.

In keeping with Bishop's (1988) distinction between Mathematics (capital M) and mathematics (small m), we have argued that the teaching and learning of mathematics must always be culture- and value-laden. In particular, after drawing attention to the increasingly widespread use of the distance mode in mathematics education programs, we disputed claims that quality mathematics materials produced for mathematics programs at one institution can be translated, virtually intact, to other places without compromising the quality of the educational experiences of learners. To support this argument we made the following four points:

1. Philosophical and mathematical developments in the twentieth century—especially as these have been represented in the writings of Wittgenstein, Lakatos, and Gödel—raise the possibility that although Mathematics does not exist "out-there" (in the Platonist sense) as an absolute form of knowledge, it does exist as a socially constructed body of knowledge (Ernest, 1991). In that sense, mathematics can be regarded as part of what Evers and Walker (1983) called the "seamless web of knowledge." Although the agreements represented by the existence of (capital M) Mathematics tend to overshadow the practice of (small m) mathematics in many societies, mathematics education cannot avoid, and should not try to avoid, being heavily influenced by cultural forms.

2. The history of mathematics education indicates that a form of colonialism has always operated, and still operates, whereby representatives of "higher" cultures have believed it to be their duty to pass on (capital M) Mathematics to developing nations. This colonialism has been made possible not only by the willingness of the "higher" cultures to pass on their knowledge, but also by the desire of the developing nations to receive it.

3. The practice of colonialism in mathematics education, be this conscious or

unconscious, has resulted in the downgrading of local forms of mathematics, and a consequent acceptance of rote teaching and learning of Mathematics.

4. Mathematics curricula should be localised to the extent that mathematics education around the world can be enriched by the variety of local mathematical forms which are part of the learners' personal worlds.

The relevance of radical constructivist and situated cognition research for mathematics education curriculum development and research in the Asia-Pacific context was also discussed. In particular, the chapter summarised several recent attempts to develop mathematics teaching approaches and mathematics curricula which take greater account of prior knowledge and the personal worlds of individual learners.

5 Research and the Mathematics Classroom: The Move Towards Action Research

Research Versus Policy

It is well known that the findings of mathematics education research have had little impact on how mathematics is taught and learned in schools. Traditional "chalk and talk" approaches in which textbooks and pencil-and-paper tests feature strongly, continue to be widely used (Garet & Mills, 1995; Gregg, 1995; Olssen, 1992). Often, university-based researchers have unfairly blamed "teacher inertia"[45] for the fact that schools and school systems have not been greatly interested in developing programs which incorporate the main messages from their research findings.

In fact, irrespective of whether they have had a sound research base, approaches recommended and funded by Ministries of Education have been likely to have a greater short-term impact in schools than approaches based on findings from university research studies.[46] Perhaps this has been the case because teachers have felt more or less compelled to do what they were being told to do by their employers. An obvious ethical dilemma arises when teachers are given leave to attend professional development programs at which government education policies are presented and emphasised. If, subsequently, these same teachers are regularly visited by Ministry consultants appointed for the purpose of assisting

45. The term "teacher inertia" is unfair to teachers because it suggests that it is teachers who have resisted desirable changes. It could be argued that a more accurate explanation of the failure to achieve significant change is that the quality and structure of professional development programs designed for the purpose of assisting teachers to achieve change in schools have been inadequate.

46. Ideally, of course, approaches recommended and funded by Ministries of Education would have a sound research base. However, in systems driven by government policy, this is often not the case. Thus, for example, although the outcomes-based-education (OBE) policies introduced in the United Kingdom, Canada, and Australia over the past decade did not have a strong mathematics education research base, they have been put forward as part of the "economic rationalist" education agenda of governments (Ellerton & Clements, 1994).

them to develop programs and methodologies in line with government policies, then almost inevitably the teachers will feel pressured to comply—or, at least give the impression that they are complying—with "official" directives.

The danger with such a state of affairs is that when policies change, or funding is withdrawn, teachers go back to doing what they did beforehand. In this sense, what has been called the "inertia" effect might be more accurately termed a "pendulum backswing" effect (Glidden, 1996). One can hardly blame teachers who do not "own" change initiatives for choosing to return to their "comfort zone."

Enabling Teachers' Voices to be Really Heard

Apple (1995) has argued, persuasively, that despite all of the rhetoric about teaching and professionalism, and about enhancing teachers' power and respect, the reality of many teachers' lives bears little resemblance to the rhetoric. According to Apple (1995):

> Rather than moving in a direction of increased autonomy, the daily lives of teachers in classrooms in many nations are becoming increasingly controlled, increasingly subject to administrative logics that seek to tighten the reins on the processes of teaching and curriculum. Teacher development, co-operation and "empowerment" may be the talk, but centralization, standardization and rationalization are the tendencies. ... Teachers are, increasingly, being expected to do what they're told. In school system after school system, teaching methods, texts, tests, and outcomes have been taken out of the hands of the people who must put them into practice. Instead they have been legislated by state departments of education, state legislatures, and central office staff, who in their attempts to increase "the quality of educational outcomes" are often markedly unreflective about the latent effects of their efforts. ... The most obvious result is the de-skilling of teachers. (p. 333)

What Apple has said of teachers in the United States of America is, we would maintain, applicable to teachers in all Australian states and territories and in New Zealand (Edsall, 1996; Ellerton & Clements, 1994; Neyland, 1996; Zadkovich, 1996) and, no doubt, in some other nations, too (for example, the United Kingdom—see Bowe, Ball, & Gold, 1992; Donald, 1992).

A similar point could be made with respect to the effects of the application of international aid money (such as that often available through UNESCO, the World Bank, or the Asian Development Bank) on mathematics education practices in schools around the world. Critics of the educational assistance offered by First World donor agencies and the interests they represent have argued that these organisations often adhere to conservative Western ideologies, leading to the funding of inappropriate and ineffective research projects (Kitchen, 1995).[47] Teachers are not involved in decisions about which projects should be funded, but they are expected to implement recommended programs without asking too many questions. These critics have called for the funding of projects which enable the voices of teachers and other educators working in the field to be heard.

47. Note that although we understand the point of view represented by these criticisms, we do not necessarily agree with them. For example, in writing this book, which is being published by UNESCO, we have not been pressuredto represent the views of UNESCO, or of Australian mathematics education, or indeed of any organisation or group.

There is a major need for the funding and carrying out of mathematics education research which genuinely takes account of the needs and ideas of teachers of mathematics. In short, Teachers' voices need to be heard. As Kitchen (1995) has stated:

> Another important reason to study mathematics pedagogy from a Third World perspective is simply to support teachers working in difficult conditions. One means to back our Third World colleagues is to study their work, and to share with others the battles they have fought and the struggles that they have endured. This study can serve the purpose of demonstrating to others working in similar conditions that they are not alone. (p. 2)

The ambivalence of practising teachers towards the results of mathematics education research, reflected by their general unwillingness to accept the advice of self-styled "experts" (who are often perceived as working in "ivory towers" far removed from the pressures of the classroom), has been one of the factors which has resulted in the development of different approaches to the professional development of teachers.

It is generally accepted by action researchers in the 1990s that action research in schools is carried out by teachers and is *not* something which is done for teachers, or on teachers and their classrooms, by outside researchers (Cochran-Smith & Lytle, 1993). Shaughnessy (1994), in an article carrying the evocative title, "Classroom research and classroom boundaries: Blurring the boundaries," expressed the basic idea succinctly:

> When teachers take on the role of researchers, classroom research moves from the teachers being the object of research to a proactive state in which teachers themselves identify questions, collect and analyze data, and share findings with their colleagues. In this manner, research by teachers uses the voices of teachers to contribute to a broader dialogue on what is known about teaching. (p. 644)

Action research theory and practice have moved a long way from Elliott's (1978) suggestion that action research "may be carried out by the teachers themselves or by someone they commission to carry it out for them" (p. 355).

However, in making this plea for research which enables teachers' voices to be heard, we hasten to add that in our own nation, Australia, it has been argued (Bates, 1994; Ellerton, 1995a) that the Federal and State Ministries of Education have been funding "research" by friendly researchers who have produced reports which make selective use of "teachers' voices" and which, almost inevitably, express support for government education policies.

This raises the question of the type of research which is needed to enable teachers' voices to be *really* heard. Furthermore, the issue of *who* decides which teachers' voices are to be heard must not be overlooked.

Action Research and the Ownership of Change

One "new" approach,[48] which is usually referred to as "action research," is based on the premise that if teachers themselves are fully involved as voluntary participants in education research projects, then the research findings are more

likely to be integrated into practice (Baird, Mitchell & Northfield, 1986; McTaggart, 1991; Richardson, 1994; Robinson, 1989; Warner, 1996).

Action research has become a *cause célebre* in some nations of the Asia-Pacific region. For example, during the period 1994–1996 the Malaysian Ministry of Education funded about 500 action research projects in Malaysian schools, as part of its Program for Innovation, Excellence and Research (PIER) (see Madzniyah, Hussin, Choo, Embong, Sani, Idris, & Kim, 1995). Education leaders in all Malaysian States have received training in the theory and practice of action research, and all schools have been invited to submit their own action projects for possible funding from the Ministry. Many of the funded projects were submitted by teams of mathematics teachers. In fact, over the past decade many mathematics teachers in the Asia-Pacific region have been involved in action research projects, and as a consequence some have changed the ways they teach and have improved their classroom environments (Ellerton, Clements & Skehan, 1989; Mousley, 1992; Robinson, 1989).

Discussions on the role and potential of action research for improving schools have been hampered, however, by the fact that the expression "action research" is being used by educators all over the world to mean many different things. It is important, therefore, that some attempt is made to define the concept of action research in a way which is most likely to be useful in the domain of mathematics education.

One of the characteristics of action research in schools identified by Elliott (1978) was that of interpreting "what is going on from the point of view of those acting and interacting in the problem situation, e.g., teachers and pupils, teachers and headteacher" (p. 356). Elliott emphasised the importance of recognising teaching as a human endeavour rather than as a clinical process subject only to the laws of natural science.

One way of defining a concept is to list the set of essential characteristics for the concept, together with a set of characteristics which are excluded from the class of objects or events covered by the concept. It will be useful to follow this procedure in the text which follows. First, the purpose and essential characteristics of action research will be given, and this will be followed by some observations of what action research is not.

The Purpose and Essential Characteristics of Action Research

We accept the following "definition" of action research put forward by Kemmis and McTaggart (1988) as a statement of the *purpose* of action research:

> [Action research is] a form of collective self-enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social or educational practices, as well as their understanding of these practices and the situations in which these practices are carried out. (p. 5)

From this perspective, action research would normally involve much more than testing hypotheses and drawing implications from data (Carr & Kemmis, 1986).

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48. Action research is, of course, *not* new—it has been applied in various settings for at least five decades (McTaggart, 1991).

The most common image of action research is the so-called "action research spiral" (see Figure 3). Teachers looking at this cycle could be excused for thinking that they are involved in action research every day of their professional lives. After all, they *do* plan their lessons, enact their plans in their classrooms, observe the effects of their actions, and reflect on what they might do differently next time. Consistent with the notion of a spiral that has no beginning and no end, as teachers reflect on their observations and on their previous plans and actions, they develop new plans for the next lesson, the next day or the next week in the classroom. An action research "cycle" is part of a spiral which suggests the building of plans on prior actions, observations, and reflections.

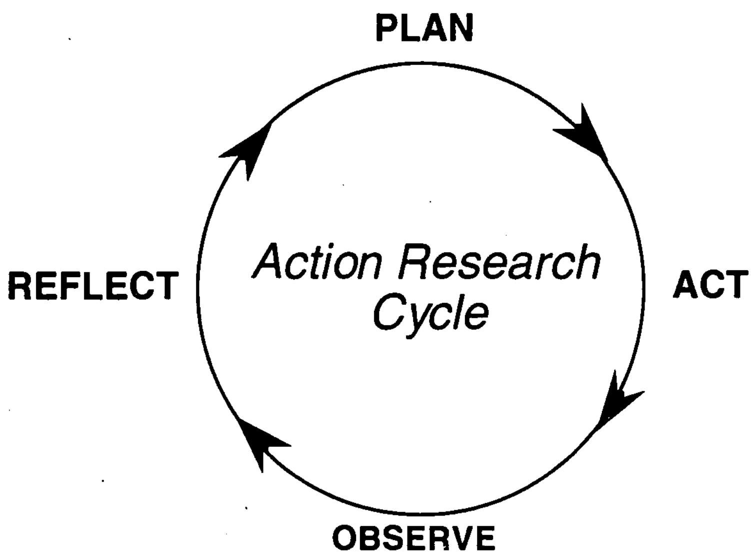

Figure 3: One Cycle of the Action Research Spiral of
Plan->Act->Observe->Reflect

To complement Kemmis and McTaggart's (1988) statement of the purpose of action research we list the following six *essential* characteristics of any education action research project:

1. *Action research involves teamwork.* Action research involves a team of committed practitioners/researchers working together to improve existing practice. All members of the team should contribute a "fair share" of the work.

2. *Membership.* Because the achievement of effective change in education does not come easily, an individual's decision to accept membership of an action research team should represent a long-term commitment. Even so, any team member should feel free to be able to withdraw from the action research team at any time.

Membership of an action research team should be voluntary, with all members regarded as being of equal status. However, after a project has begun no person ould be invited to become a new member of the action research team unless this agreed to by all existing team members.

3. *Team meetings*. Democratic team planning and reflection should take place at regularly held team meetings. Open and equal participation in discussion is essential, and the interests of "underdogs" within the team should be scrupulously protected. Decisions reached at meetings are to be mutually binding.

4. *The research agenda*. Research question(s) and corresponding methodologies should be agreed upon at team meetings. These should relate to an agreed theme of interest—the temptation to investigate a series of unrelated individual concerns, contributed by group members, is to be avoided.

5. *Documentation*. Any observations and reflections of team members should be shared at team meetings, and consensus should be reached on decisions for action. Observations, reflections and decisions for action should be documented in the minutes of the meetings.

6. *Data collection, analysis, and reporting*. Data should be collected, analysed, and reported, systematically, in a manner agreed to by the action research team.

Critical Theory and Action Research

This definition and discussion of the characteristics of action research does not draw attention to its theoretical base. As Popkewitz (1988) and McTaggart (1991) have pointed out, at the heart of action research lies critical theory, which aims to change society, in general, and education in particular, in fundamental ways.

Critical theory arose from neo-Marxist philosophies which stressed the need to develop strategies for making society more just and equitable. Critical educators, such as Carr and Kemmis (1986), who have drawn heavily from Continental philosophers such as Habermas, are convinced that formal education systems and especially schools have contributed to the development of élitist forms of education which have favoured children from middle-class family backgrounds. They see action research as the *only* way by which more equitable structures and approaches can be established. They also anticipate that the processes involved in bringing about such changes are likely to encounter resistance from privileged minorities.

What Action Research in Education is Not

According to Henry (1995), action research is *not merely*

- what teachers normally do. (In education settings action research involves teachers working together and reflecting on specific questions as they seek to improve their teaching and the conditions which influence their teaching);

- problem solving. (Action research usually involves *both* problem posing and problem solving, as well as some consideration of whether a problem as originally stated by others is *really* the problem which needs to be investigated);

- *informal* research projects in which teachers reflect on their practice. (Those engaged in action research can, in fact, choose whatever instruments and methods of analysis they deem to be appropriate, including *formal* quantitative and qualitative approaches);

- any form of participatory research in which teachers collaborate. (Although action research *is* characterised by democratic, collaborative participation, it is more than this.)

Mousley (1992) has provided a useful review and summary of mathematics education research projects which were in the general category of "participatory, collaborative research," and were carried out in Australia and New Zealand over the period 1988–1991. Although all of the studies Mousley cited would belong to the general category of participatory studies involving the collaboration of two or more interested parties, most of them would fit into one of Henry's (1995) four categories, and would *not* be examples of action research as we have defined it in this chapter.

False Advocates of Action Research in Education

Action Research, Official Policy, TQM, and Outcomes-based-education

There is an increasing tendency for governments to formulate education policies and then to provide funding for collaborative projects in which paid consultants are given the responsibility of involving teachers and schools in assisting the implementation of government policy. In such projects, teachers in co-operating schools (often called "trial schools") are supported by consultants, and the schools receive additional funding to meet expenses.

Although there is nothing inherently wrong with this tactic, we would point out that attempts to achieve change through such "top-down," "authority" models (Marsh, 1994) should not be included within the ambit of action research. We cannot agree, for example, with Bonser and Grundy's (1995) argument that the imposition of outcomes-based approaches on Australian teachers by government authorities has allowed the teaching community to move towards becoming a more reflective, collaborative, profession in which a fundamental rethinking of teachers' roles is likely to occur. Is it realistic to assume that the mere adoption of outcomes-based approaches across whole school systems will, of itself, generate reflective practice and healthy collaboration and cooperation among teachers? After the trial period is over, will the level of funding provided for trial schools subsequently be offered to all schools?

There is evidence from the United States of America, the United Kingdom, and Australia that once the initial impetus for the introduction of outcomes-based approaches in schools dissipates, and additional funding disappears, the additional expectations on teachers and schools are counter-productive (Groundwater-Smith, 1993; Stake et al., 1993, 1994; Truran, 1993). In Australia and New Zealand, despite the large amount of government funding allocated to OBE approaches during recent years (Ellerton & Clements, 1994: Neyland, 1996), many teachers are rejecting the underlying assumptions of OBE and are annoyed by the expectation that they should conform to policies which put pressure on them to teach in ways which are antithetical to their own education philosophies (Edsall, 1996; Lanyon, 1995; Usher & Edwards, 1994).

Some supporters of OBE policies believe that "action research" procedures have assisted the introduction of OBE approaches into Australian schools. The tendency to invoke the name "action research" to support government-imposed :hemes is evident in the following comment by Sachs and Logan (1995), in a paper

entitled "Using school development planning as a vehicle for professional development":

> School Development Planning (SDP) is an externally imposed procedure nested within a set of management strategies used by successive governments to restructure Australian school systems. We argue that SDP offers the potential to combine two powerful forms of professional development, namely, narrative inquiry and action research. Narrative inquiry structures the social interaction of planning. Action research provides a process for reflecting on and recording planning and its outcomes. Together, we maintain, these two forms of professional development provide the means for individual and collective creation and validation of professional knowledge and in so doing contributes to the dialogical development of teachers' learning. (p. 3)

There is a tension raised by this statement. Presumably Sachs and Logan believe that the imposition of "an externally imposed procedure nested within a set of management strategies used by successive governments to restructure Australian school systems" can be reconciled with an action research framework. We do not accept that viewpoint, but recognise that much depends on how "action research" is defined.

Education academics find themselves in an invidious position with respect to the top-down approaches which are characteristic of many government initiatives in education. By agreeing to become consultants to projects which derive from policies which are not consonant with the findings of education research, well-meaning education academics are in danger of compromising their professional integrity (Bates, 1994; Ellerton & Clements, 1994)?

We would maintain that education researchers who suspect that government education policies are out of line with the results of the best education research have a responsibility to say so. They should resist offers to become involved in so-called "action research" projects which, a cynic might say, are part of government strategies to impose policies which are not in line with the findings of independent research. Teachers and researchers should not be used as means for providing respectability for misinformed policies. This is especially important at this time when teachers are likely to be punished for speaking out against government education policies. As Zadkovich (1996) has stated, "we need lively and challenging debate, not timidity, in the face of educational faddism (or in some quarters, educational fascism)" (p. 16).

One reply to the assertions in the previous paragraph might be that teachers can adopt *genuine* action research procedures in their efforts to cope with the extra demands imposed on them by their involvement in practices deriving from new policies. This seems to be the argument presented in *Collaborative action research: Working together for improvement*, a joint publication of the New South Wales Department of School Education and the Australian Council for Educational Administration, NSW (1996), which featured a Tasmanian action research project. This centrally initiated and funded project was concerned with the theme "Educative accountability policies for locally managed schools," and schools were accepted into it on the basis of proposals submitted by stakeholders ("parents, teachers, principals, and DEA officials"). In such circumstances, if one is to be true to the principles of action research, then action research teams might need to devise strategies for ensuring that the data they collect is their own, and is not used,

without permission, for "advertising" purposes. They might also need to devise strategies for coping with school and system demands placed on them by over-zealous school administrators and Ministry consultants and officials.

In fact, like Prawat (1996) and Usher and Edwards (1994), we regard the OBE version of how Total Quality Management (TQM) ideas can be applied in education as essentially incompatible with the ideals of action research. The *sine qua non* of OBE is to be clear about the ultimate destination. According to this perspective, which underlies TQM, good administration, like good teaching, starts with a clear definition of desired results—only then can one engage in the means-end analysis necessary to bring rationality to the process. However, as Prawat (1996) points out:

> The learning community approach to reform, and teaching, challenges this basic assumption. Rather than think of either reform *or* teaching as the pursuit of a fixed agenda, a well-defined 'course to be run', subject to external adjustment at various points, it is more appropriate to conceptualise the process as a kind of cross-country exploration of the new terrain. (p. 106)

Zadkovich (1996) calls for the resurrection of a culture that actually allows teachers to disagree and debate and contest policies, issues, ideas and practice in contemporary education. Unless those conditions are present, it is hard to imagine how action research can do anything but tinker at the edges of education change.

Action Research and its Status in the Education Research Community

Some "advocates" of action research seem to believe that it does not have equal status with "formal" research studies in education.

There is a commonly-held belief, even among professional educationists, that although action research is fine for school teachers, it is not really appropriate—in the sense that it is not sufficiently rigorous—for university education researchers. Those who subscribe to this view tend to argue that the findings of action research are less reliable than those of formal research, and that an action research methodology should not normally be used by masters- or doctoral-level education researchers. There is a tacit assumption that the only *genuine* research results are those which are *generalisable* to systems of education, and that action research produces results which are pertinent only to a single classroom or school, and are therefore of limited use and interest.

Although such sentiments are commonly expressed, we believe that they reflect a lack of understanding of the principles and origins of action research. In fact, the results of action research projects can assist the difficult processes of theory generation and theory evaluation..

There is such a diverse range of languages and cultures in the Asia-Pacific region that research based on outsiders' grand theories is likely to take insufficient account of important factors influencing what is under investigation. Such research is likely to produce "findings" which do not match the needs and aspirations of the main stakeholders.

Action research is not atheoretical, however. Those who engage in action research tend to be more interested in theory-generation and, subsequently, theory-modification, than in the application of universal "grand theories." For action researchers, theory is developed from reflections on observations of actions; in action research there is less of a tendency than with traditional research to make observations "fit" theory. This lead to a consideration of how action research and traditional education research are related.

In the context of this book, it could be argued that it is particularly important that action research methodologies be increasingly employed in mathematics education research throughout the Asia-Pacific region.

Action Research and Traditional Education Research

How, then, is action research related to education research? Traditionally, it has been assumed that most education research is either of a historical or philosophical nature or is carried out in a classroom or school context by researchers who normally work outside the classroom. Such a view has located the day-by-day planning, decision making, teaching, and evaluation processes with practising teachers, but research into such activities has been something "done" by "outside experts." Action researchers regard this traditional view of education research as separatist and élitist.

Education research, as seen by advocates of action research, has different dimensions from traditional education research. Action researchers maintain that the day-by-day work of teachers provides a legitimate data base for worthwhile research. From their perspective, teachers and administrators continually make decisions based on data. For example, suppose William, a very sensitive student who has often failed mathematics tests, provides a very insightful mathematical solution to a complex question related to athletics which, the teacher knows, is a special interest of William's. The teacher's response to William's solution will be determined by the teacher's interpretation of all available relevant data. Typically, teachers respond immediately and spontaneously to numerous issues such as this throughout a normal working day.

However, action researchers do not regard such immediate and spontaneous responses as belonging to the realm of action research unless the previously mentioned characteristic features of action research are also present. These features include the requirement that any action research project must involve a group of participants who are seeking to improve their own practice. There are also the features of collaboration, democratic decision making, and record keeping referred to above.

The traditional image of education research is research which involves carefully designed quantitative and/or interpretive studies. With these traditional approaches, the research questions, and the instruments for collecting and the methods for analysing data, must be made explicit before data collection begins. Detailed, often highly statistical or complex qualitative methods are used to analyse data.

By contrast, action research is a methodological framework or philosophy whereby participants may or may not choose to use aspects of some of the traditional education research approaches. As stated by Kemmis and McTaggart (1988), action research "is not research done on other people. Action research is research by particular people on their own work, to help them improve what they do, including how they work with and for others" (p. 22). Kemmis and McTaggart (1988) went on to say:

> Action research is not just about hypothesis-testing or about using data to come to conclusions. It adopts a view of social science which is distinct from a view based on the natural sciences (in which the objects of research may legitimately be treated as 'things'); action research also concerns the 'subject' (the researcher) him or herself. Its view is distinct from the methods of the historical sciences because action research is concerned with changing situations, not just interpreting them. Action research is a systematically evolving, a living process changing both the researcher and the situations in which he or she acts; neither the natural sciences nor the historical sciences have this double aim. (p. 22)

An attempt to clarify the relationship between action research and more traditional education research paradigms is shown diagrammatically in Figure 4 (which is taken from Ellerton, 1995b). This figure suggests that traditional education research, which is aimed at mapping, understanding, and modifying existing practices, is essentially different from action research, for which the fundamental aim is to improve education practice.

However, because successful planning for improvement often depends on gaining a better understanding of existing structures, most action researchers are willing to draw on the methodologies and findings of traditional education researchers. In that sense, action research and traditional education research are complementary. There is little point in teachers engaging in action research projects which, after much effort, merely "reinvent the wheel." Action research can, and should be informed by the findings of traditional education research. That is why it is often a good idea for action research teams to include at least one person who has a good knowledge of the findings of traditional education research with respect to areas of concern which are likely to be the focus of study by the team.

Figure 4 suggests that traditional researchers should also be willing to learn from and draw upon the findings of action research. However, clearly not all traditional education researchers are willing to accept action research principles and practices, and not every action researcher feels the need to draw heavily from traditional education research findings and methods.

Note that there is no intention to suggest that all forms of education research are covered by the labels included in Figure 4. For example, grounded theory is not mentioned, but grounded theory approaches (Strauss & Corbin, 1990) have been widely used in the last decade by education researchers who believe that, at this stage in the history of education, theory-generating research is more important than theory-driven research. There are many other labels—such as post-structuralist, post-modernist, feminist—which would be included in any comprehensive list of contemporary education research paradigms, but this chapter has attempted to link action research with the more traditional education research paradigms.

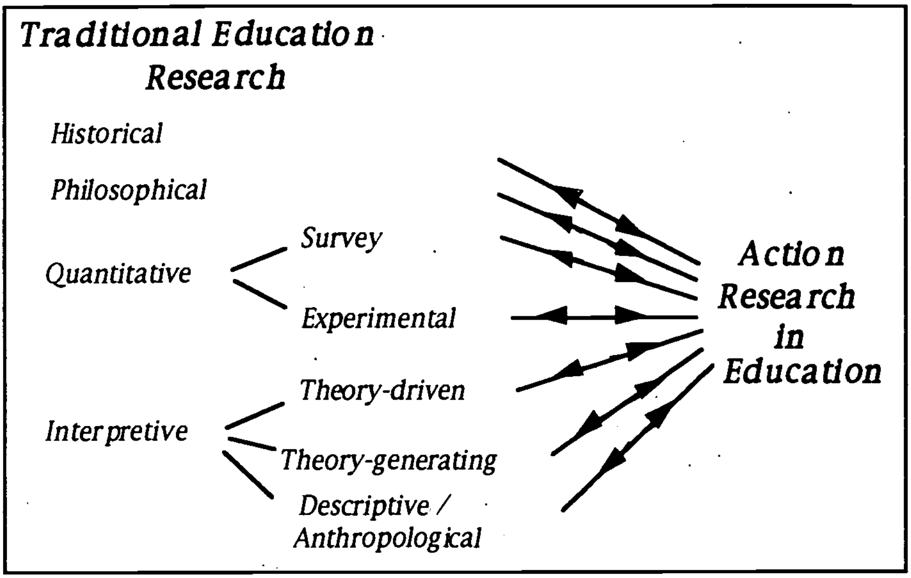

Figure 4: A possible relationship between action research and traditional education research (from Ellerton, 1995b).

Perhaps the most important point to note from Figure 4 is that action research should never be regarded as just "another" education research approach. A logical inference from the diagram is that much of education research is potentially of little value unless it is linked to, or embedded in, an appropriate action research context. A further inference is that action research can gain if it is informed by the results of traditional education research.

In the section which follows, five case studies will be used to help illustrate the points raised both in the description of action research, and in the discussion of the relationship between action research and education research.

Five Case Studies: Action Research in Action

Each action research project is unique. Its nature is defined by those who have designed it and are implementing it. They are in control of what is being done, and make decisions on what will be done in the future. Despite the unique character of any single action research project, sharing what others have done can be a powerful way of demonstrating different aspects of action research.

The following five summaries of action research projects are presented, therefore, to illustrate how teachers, from a range of contrasting contexts, identified problems associated with their own practices. They then developed and implemented strategies for solving these problems.

1. The Institute for Educational Development of Aga Khan University

In January 1994 the Aga Khan University in Karachi, Pakistan, introduced a professional development program aimed at assisting leading teachers to collaborate with their colleagues in the schools, for the purpose of upgrading the quality of instruction in the schools. Action research principles have been used throughout the project (Bacchus, 1995a; Kanu & Wheeler, 1995). That, in itself, is interesting because, according to Kanu and Wheeler (1995), most teachers in Pakistan lack detailed knowledge of the content of subject areas and, usually, their teaching is not characterised by a spirit of critical reflection.

A noteworthy feature of the project is that all the teachers in the action research team were enrolled as graduate students in teacher education. The first aspect of the project involved the teachers in a critical re-examination of their conceptual frameworks. This provided an essential preparation for their future role as participants in school-based action research teams which would seek to deconstruct and reconstruct teaching environments in the schools.

In this project, the leading teachers enrolled in the Aga Khan University graduate program made the decision to regard each participating school as an entity in its own right. According to Bacchus (1995a):

> It was assumed that schools were in some ways like "total" institutions with cultures of their own, including a fairly closed network of interrelationships. Therefore upgrading one or two teachers and sending them back to their schools to bring about change would not, by itself, be very effective. It was therefore felt that we should work with schools as a whole—with the head teachers, with other classroom teachers, with school supervisors and others—to help create a culture that was supportive of the change which the professional development teachers were likely to initiate on returning to their schools. (p. 8)

Each participating school, therefore, was allowed to develop its own research agenda. The active support of head teachers and school managers in schools participating in the program was sought and obtained (Kanu & Wheeler, 1995).

Bacchus (1995a) reported that, at first, the leading teachers had attributed deficiencies in their schools to a lack of adequate equipment. Gradually, however, they, and the teachers in the schools, began to realise that despite inadequate equipment, major improvements could be achieved if teachers were willing to commit themselves to the project. Although at first this had a disequilibrating effect on the teachers, after a period of reflection and adjustment, they began to recognise that often their education rhetoric did not match their own practices. Bacchus (1995) reported the following observations from leading teachers:

> After one of our sessions in the classroom we sat together and praised ourselves for having been successful—once again blowing our own trumpets in the name of critical reflection. In essence, as yet another [leading teacher] said "We are talking the talk but not walking the walk." Another observed, "I tell you frankly, most of the [leading teachers] and some faculty members say they value critical thinking. But in their actions they show they do not. They have not reconciled the idea of critical thinking with their desire to hold on to their own traditional values ... [However] for all their negativity and professed disillusionment, the [leading teachers'] perspective at this point marked an important turning point in their learning process.

> Initially they had difficulties with internalising the deeper implications for themselves and the overall way in which they had perceived the learning/teaching process of the new ideas and practices to which they had been exposed in the course. (p. 12)

Bacchus's analysis revealed that over a period of 18 months, the action research teams in the schools began to question their practice in fundamental but healthy ways.

Yet, the leading teachers were enrolled in graduate programs at Aga Khan University. Traditionalists might claim that there are good reasons why research dissertations should not be based on participation in action research projects. They might point to a project being "owned" by the action research team and not just the person writing the dissertation. Even the design of the research, and the mode of analysis has to be agreed upon by the team. Furthermore, the team will have been involved in decisions on how the project is to be reported.

We believe that the Aga Khan University approach represents a positive step towards higher degree research in the field of education having immediate and obvious relevance.[49] The decision to accept action research projects for graduate studies represents a challenge to universities around the world.

2. *Key Group—A Model for the Professional Development of Primary School Teachers*

Groups of three teachers from each of a number of schools in the Australian state of Victoria were invited to form teams, and to provide mutual support for each other. A mathematics consultant from the State Ministry of Education was assigned to each group, with the consultant acting both as a member of the group and as someone expected to provide ongoing support from a different perspective. This group of four was called a Key Group.

A three-day live-in "planning" conference was held for 18 Key Groups. At this conference, each Key Group "reflected on its current practice, celebrated its successes, decided on some aspect of its mathematics teaching which it wanted to improve, and devised a plan of action for setting about it" (Robinson, 1989, p. 276). Small grants were awarded to each Key Group, and each was also given teacher-release time to enable the teachers to carry out their action plan.

Key Group exemplified professional development through what Robinson (1989) called the "empowerment paradigm." It recognised the central role of teachers, and assumed that the teacher was an expert on education and on learning. It also took the teacher's current practice as the starting point. Robinson emphasised the importance of the attitude and motivation of the outside agent involved. He pointed out that in the past some change agents had used their role to reinforce their own self-image, by "actually causing change to occur in clients ...

49. Of course, there are many *other* faculties of education where action research projects are accepted for higher degree dissertations. (For example, the Faculty of Education at Deakin University (Victoria, Australia) is well known for its advocacy of action research, and for the emphasis in its graduate program on action research.)

irrespective of the needs or wishes of the client" (p. 279). This approach was actively discouraged with Key Group.

According to Robinson (1989), it has often been the case that enthusiastic change agents, who had been outstanding teachers themselves, became "evangelists" who accepted the role of telling everyone about how they believe things should be done. By contrast, Robinson said, caring facilitators "encourage each group member to recognise and take responsibility for the choices underlying that member's own behaviour. The caring facilitator helps to define choices; change is the participant's privilege" (p. 280).

For Robinson, the "empowerment paradigm," as exemplified by Key Group, represented a proven alternative to the traditional hierarchical management paradigm, which tended to regard people (and in this context, teachers) as objects to be managed. Interestingly, Robinson (1989) maintained that the ideas of Michael Fullan—often regarded as the doyen and most prolific writer of the new "management of change in education" theorists—were more in line with management paradigms than with genuine action research approaches.

Ten years after the beginning of Key Group in Victoria, the concept is thriving, with large numbers of schools participating in a "Maths in Schools" project, which is jointly sponsored by the Mathematical Association of Victoria and the Faculties of Education in all universities in the State of Victoria (Ferguson & Montgomery, 1994). Two to four staff members were nominated by each university to work in teams. In 1994, twenty university staff were matched with 20 school project teams. Often the project being investigated in a school matched the interests of the university staff member(s) in its team, but this was not always the case for "the intention was to enlist a support person, not a visiting expert" (Ferguson & Montgomery, 1994, p. 42)

3. PEEL—A Project for Enhancing Effective Learning

The major aim of PEEL, which began in 1985 at Laverton High School in Victoria, Australia, was to improve the quality of school learning and teaching. The approach adopted by the group of teachers and consultants who initiated the project focused on giving students training so that they would become "more willing and able to accept responsibility and control for their own learning (Baird & Mitchell, 1986, p. ii).

Baird (1994) has listed the following agreed-upon objectives of PEEL:

1. To foster effective, independent learning through training for enhanced metacognition;
2. To change teacher attitudes and behaviours to ones which promote such learning;
3. To investigate processes of teacher and student change as participants engage in action research;
4. To identify factors which influence successful implementation of a program which aims to improve the quality of students' learning. (p. 12)

From the outset, it was recognised that achievement of the changes envisaged would require time and patience on the part of all involved. Weekly meetings of the PEEL group were held, with regular informal contact occurring between teachers and "outsiders" involved in the project.

One of the strengths of PEEL was (and still is) the relationship between all participants involved in the action research. In their Preface to the book *Learning from the PEEL Experience*, Baird and Northfield (1995) described how, from the beginning, "PEEL employed a method of group-based action research where teachers, assisted by the tertiary members, acted together to research classroom teaching and learning" (p. ii). Baird and Northfield reinforced the notion of action as meaning that the participants who are involved in the process are, in fact, those who carry out research into the process.

According to Baird and Northfield (1995), action research in PEEL has involved the participants in confronting and challenging the participants'

> closely-held attitudes, perceptions, conceptions and abilities relating to the nature of learning, the profession of teaching, one's own worth and success, and personal satisfaction and fulfilment. The process of change is often tiring, sometimes upsetting, sometimes rewarding, and always challenging. (p. ii)

Thus PEEL has involved the participants in actively making substantial changes to their views of and strategies for teaching and learning. However, it is through these processes of change that one of the most important resources for change has been tapped—the teachers themselves.

One of the major aims of PEEL has been to improve the quality of the discourse in the classrooms of participating teachers. Over time, data which indicate that this aim is being achieved have been observed in PEEL classrooms, with a gradual shift occurring from interactions characterised in the first column shown in Table 3 to those listed in the second column. In describing the pairs of statements in Table 3, Mitchell (1995) pointed out that the "from" and "to" statements were not intended to be alternatives which allowed no middle ground, but that PEEL has achieved "a substantial shift towards the right-hand descriptors for most teachers" (p. 75).

Clearly, a project such as PEEL would be of less interest and value if it could not be sustained in other schools and contexts which involved different people. From the initial involvement in PEEL of some staff from a single secondary school, PEEL has spread in less than 10 years to between 10 and 20 primary and secondary schools (Mitchell & Northfield, 1995). Two books which document the Project have been published (Baird & Mitchell, 1994; Baird & Northfield, 1995), and a monthly newsletter ("PEEL Seeds") sustains contact and discussion between those involved, as well as initiates the sharing of ideas and reports.

Interestingly, in the early years of PEEL, teachers of mathematics were reluctant to be involved. However, as the influence of the Project expanded, teachers of mathematics in a number of schools have formed action research teams. The November 1995 edition of "PEEL Seeds" contained reports from meetings of two of these "mathematics" teams (as well as reports from teams working in other curriculum areas). Mathematics teachers are now much more likely than they were a decade ago to want to be involved in across-the-curriculum PEEL support teams (Horne, 1995).

Table 3
List of General Teacher Behaviours (from Mitchell, 1995)

FROM	TO
1. Asks closed questions requiring a predetermined short answer.	Regularly asks open questions calling for one or two sentence answers where there is no predetermined "correct" answer.
2. Short wait time (1 second). If one student is uncertain, moves quickly to another.	Allows longer wait time (several seconds), follows up and encourages uncertain students to keep going.
3. Interested only in determining if students are giving the "right" answer. Stops listening once it is clear that the student is wrong.	Shows an obvious interest in what students are thinking. Listens carefully to what a student actually says even after it is clear that the student is wrong.
4. Accepts answers or comments (as right or wrong) at face value.	Frequently calls for students to justify or elaborate answers or comments.
5. Judges all student answers. Corrects tactfully all "wrong" answers or comments at face value.	Often delays judgment by accepting all answers neutrally, giving praise for contributions. Encourages and praises students who challenge the teacher's ideas.
6. Is determined to get from A to B in a fixed time and by a fixed route. Routinely cuts off "red herring" comments.	Flexible on route and timing and, to some extent, destinations. Collaborates with students in exploring and using their ideas and questions.
7. Sets and organises tasks in ways which put pressure on students to get on with following the instructions.	Sets and organises tasks in ways which allow for checking, monitoring and linking behaviour by the students.
8. Emphasises the learning of "what" to do much more than "why" it is done this way. Presents many rules, procedures and formulae.	Emphasises reasons and principles, encourages students to work out "what" to do on the spot. Minimises the role of rules, procedures and formulae.
9. Calls for a lot of copying or transcribing.	Copying is minimised.

4. Hunter Region Primary Principals Project

The Principal and School Development Program is a joint venture between the New South Wales Department of School Education, the New South Wales Primary Principals' Association, and the New South Wales Secondary Principals' Council. The stated aim of the Program is to promote school improvement and the professional development of principals. One of the projects under this venture has been the Principal and Teacher Rejuvenation Project undertaken by a group of 12 participants from the Hunter Region of New South Wales.

In the Hunter Region Project, eleven of the twelve participants were principals

in Hunter Region[50] schools who volunteered to be involved in the Project; the twelfth participant was one of the authors, Ken Clements, of the Faculty of Education at the University of Newcastle, who was invited by the other participants to be a facilitator for the group. Initially, eight primary school principals were involved, but subsequently three Hunter Region secondary school principals who had shown interest in the Project, were invited to join the group.

The group chose to meet in participants' schools on a monthly basis. It decided to adopt strict action research procedures. All participating group members had equal status, collaborative procedures for the working of the group were adopted, minutes of meetings were kept, common issues of concern were identified, and action plans were developed, implemented and evaluated in a mutually supportive environment. Because many of the group members were themselves experiencing considerable stress arising from their particular work contexts, group members undertook to provide support for each other whenever the need arose. The group organised public meetings which were well attended by local principals, and collected, reported and analysed data relating to questions that the group decided to investigate.

One of the investigations undertaken by the group was of particular interest. The group decided that, since stress among principals and among teachers within the principals' schools was a major area of concern, it should attempt to identify the causes of such stress and to develop strategies to alleviate these causes.

Comprehensive surveys of teachers and principals were designed and carried out by the team. Teachers were asked to provide anonymous responses to the following five major survey questions:

1. In your teaching career please identify a circumstance/s in which you felt you were rejuvenated (e.g. a change of grade, appointment as AST, an inservice course, etc).
2. Comment on what it was about these circumstance/s which gave you a sense of rejuvenation. How has the provision of leave (long service leave, leave without pay, part time teaching) helped you to be rejuvenated? Why? Why not?
3. In your teaching career please identify any obstacles that you felt may have prevented you from being rejuvenated. Comment on how these obstacles prevented rejuvenation. (If referring to people please do not identify by name, but the type of person)
4. (a) Whose responsibility is it to rejuvenate teachers?
   (b) Why?
5. To help with possible rejuvenation strategies what would you like to see occur that would instigate rejuvenation? (Besides personal, what else has happened that has impressed you?) You may like to comment on conditions that need to be present in schools for rejuvenation to take place.

The group decided on how the data should be analysed. It was agreed that simple frequency counts were all that was required. The group members worked together as a team in carrying out the analysis, and three major common factors which contributed to stress within the cultures of the schools involved were

50. Note that under a New South Wales Ministry of Education restructure in 1995, this Region no longer exists.

identified. It was agreed, on the basis of the data, that the main sources of stress were, in order of importance:

1. Demands imposed by the bureaucratic system of education (e.g., a new mandatory curriculum, new prescribed assessment procedures, large class sizes, inadequate salaries);
2. Forces and pressures associated with individual schools (e.g., parental pressures, inadequate equipment, shortage of appropriately qualified teachers);
3. Work practices and personal characteristics of individual teachers and principals (e.g., idiosyncratic behaviour, difficulty in maintaining class control).

A 44-page report on this aspect of the Project (Hunter Region Principal and Staff Development Program, 1995) was prepared. All group members participated in the development of the document, and unanimously endorsed it. The document included a summary of actions which were needed to achieve rejuvenation of teachers and principals. System-level, school and individual factors contributing to stress of teachers and principals were identified. The report included recommendations with respect to professional development strategies, and described work which had been done towards implementing these recommendations.

It is interesting to note that, in the second half of 1995, the regional structure of the Department of School Education was replaced by a structure with smaller "districts" instead of "regions." Group members found themselves located in different districts and subject to different administrative formalities. At a group meeting in October 1995, members recognised that the collegiality and network support provided by the Project had become an integral part of their own functioning as principals. They decided to continue meeting as a group, regardless of the change in administrative structure, and despite the fact that no external funding would be available.[51]

Of course, the problem of stress among school principals, and burn-out among school teachers is not confined to the Hunter region of New South Wales. It is interesting, for example, that in May 1993 the Singapore Teachers' Union decided to investigate the same kind of problem in Singapore. The Union decided to commission the services of a Senior Lecturer in Psychology at the University of Singapore, Dr Elizabeth Nair, to consider ways and means of alleviating the levels of stress experienced by teachers in schools. Dr Nair initiated a large-scale action research program, the findings of which have been fully documented (Singapore Teachers' Union, 1995).

5. *The Thinking in Science and Mathematics (TISM) Project, RECSAM*

The *Thinking in Science and Mathematics* (TISM) Project was first conceptualised in 1987 by Mr Tan Sean Huat, a Research Officer at the Regional Centre for Education in Science and Mathematics (RECSAM) in Penang (Malaysia). Tan had

51. Throughout the Group's operation the Ministry of Education has provided only minimal funding to support Group activities. Members of the Group learned to value the professional and personal benefits associated with active involvement in the network.

become interested in starting collaborative action research projects with school teachers of mathematics and science and, after discussions with visiting RECSAM consultants from Canada, Australia, and New Zealand, he and three interested colleagues at RECSAM developed a proposal for the project. This proposal was approved by RECSAM's Co-ordinating Committee in December 1988.

RECSAM staff—particularly Tan Sean Huat and Sim Jin Tan—were involved in the TISM Project and worked with staff from a secondary school in Penang. Dr John Baird, of Monash University, who had had considerable involvement in the PEEL Project, served as an overseas consultant. TISM carefully identified many separate phases for its action research projects: Bookwork, Conceptualisation, Interaction, Literature Search, Research Proposal, Identification of Project Sites, Consultation, Documentation, Interfacing, Planning/Training Workshops, Research Proposals of Teachers, Workshops, Implementation of Research Programs, Meeting RECSAM Collaborators and Consultants, Completion of Implementation Phase, Consolidation of Project Report, Publication of Project Report, Seminars, and Decision on Extension of Project. Consensus was also reached on a suitable time-frame for action (for details, see Tan & Marinas, 1990; Tan & Sim, 1991).

Weekly TISM meetings were held, with RECSAM staff travelling to the Project site; the school Principal allowed TISM team members on his staff one hour each week to attend meetings. These meetings were chaired by the TISM site co-ordinator, who was a staff member at the school. RECSAM staff acted as minute secretaries. Each meeting began with the reading of the minutes of the previous meeting, and decisions or actions with respect to on-going issues were reported.

The teachers expressed a desire to know more about constructivist ideas in education, and especially about how these might affect science and mathematics teaching and learning in their school. This therefore became a special theme of the TISM Project, and TISM members worked enthusiastically and collaboratively in their efforts to embrace the major tenets of constructivism.

Although, in the mid-1990s, Tan Sean Huat and Sim Jan Tan were no longer staff members at RECSAM, the influence of the TISM Project remained. Tan Sean Huat and Sim Jan Tan had moved to senior positions in secondary schools in Penang (Malaysia), but both were still involved in TISM—which, like PEEL, had spread its influence to many schools. When the authors of this book visited RECSAM in September 1995 they were invited by Tan Sean Huat and Sim Jin Tan, to attend a meeting of teachers in Penang who were involved, or at least interested in being involved, in action research projects. Sixty teachers attended.[52]

52. TISM was not the only successful RECSAM-sponsored action research project. Another highly successful project was the Teaching-Learning Project (TLP), which produced a range of Technical Reports in which activities of action research teams were documented (see, for example, Ferrer, Leong, & Liau Tet Loke, 1989; Leong & Ferrer. 1991; Sulaiman & Menon, 1990).

Conclusions

Not all action research projects are successful. In concluding this chapter, it is useful to attempt to identify some key features which are associated with the successful action research projects which have been summarised. Members of the Primary Principals Project group believe that the following elements contributed to the success of their project:

- The long-term nature of the program;
- The willingness of group members to make clear and succinct written statements of the problem(s) they were facing and to develop agreed strategies for solving these problems;
- The timely implementation of action plans;
- The group's careful, reflective documentation leading to a continuation of the action research cycle;
- The practical nature of the program;
- The collegiality which has been engendered;
- The collaboration which has arisen;
- In particular, the willingness of each member of the team to contribute her or his talents and time as appropriate;

These are, in fact, likely to be key features in all successful action research projects.

The reader may well ask why a whole chapter on action research has been included in this book. The answer to that question should become more apparent after the last chapter of this book has been read. In that chapter, ten propositions for future directions of mathematics education research will be presented and it will be argued that, if significant improvements in the teaching and learning of mathematics are to be achieved over the next decade, then there will need to be a thorough, even radical, reconceptualisation of current practices and policies affecting mathematics education at all levels. The role of formulating policies to achieve change should not be seen as the preserve of politicians and senior education bureaucrats, or even of mathematicians and/or university-based mathematics educators. Policy formulation, indeed the whole change process, should involve *all* stakeholders, and should enable the voices of teachers and mathematics education researchers, in particular, to be heard.

We believe that improvement in mathematics education is most likely to be achieved if genuine action research processes are adopted, funded, and valued. We also maintain that university faculties of education should not only recognise but also facilitate the involvement of staff and undergraduate and graduate students in action research projects.

The issue remains whether ministries of education, teachers, and mathematicians and mathematics educators attached to universities in Asia-Pacific nations would be willing and able to become partners in a move towards an education research culture which embraces action research. Is our advocacy of action research as a means towards achieving desirable change in mathematics education unrealistic? We think not.

The Ministry of Education in Malaysia has shown the way by coordinating a World Bank project ("Programme for Innovation, Excellence and Research") for which one of the major sub-programmes has the specific aim of working towards improving education practice in Malaysian schools through action research.[53] In Australia, during 1994–1995, the Federal Government funded 17 Round Table groups across the nation in which staff in university faculties of education and school systems combined in action research projects. In fact, there are action research projects aimed at teacher empowerment currently operating throughout the Asia-Pacific region (see, for example, Bhatanagar, 1996: Chauan,1996; Henriques, 1996; Kamaluddin, 1996). These are encouraging signs for the future.

A recent article by Bossé (1995) pointed to a basic structural similarity between the "New Math" movement in the United States of America in the 1950s and 1960s and the current movement to introduce the National Council of Teachers of Mathematics' (NCTM) *Standards* in schools across the United States. Bossé (1995) concluded that although both movements represented "valiant attempts to right the wrongs of decades of wrongdoing in the K–12 mathematics classes of America" (p. 200), both movements suffered from the exuberance of do-gooders who saw themselves as having expert knowledge. These "experts" were willing to adopt top-down methods which, they hoped, would result in the large-scale adoption of their ideas by teachers.

With the New Math, the "experts" were perceived to be university mathematicians who wanted to modernise the content of school mathematics, and ensure that logic and rigour characterised the new programs; with the NCTM *Standards* movement, the "experts" are influential tertiary mathematics educators who think that constructivist theories and practices hold the key to achieving a revolution in the nation's mathematics education programs. They too, according to Bossé (1995), are happy to use top-down procedures as a means to an end.[54]

Ethical Issues

Bossé (1995) argued that "in order for the *Standards*, and any other educational reform to be successful, the scene must be set in which classroom participants evolve with curricular reform and not continually after it" (p. 201). He continued:

> When teachers are considered truly professionals and they become part of the evolutionary process, then the current game of chase will end. This will however necessitate encouragement from curriculum developers, reformers, and policy makers. Teachers must be encouraged to improve the *Standards*, and not merely subscribe to the *Standards*. (p. 201)

We would want to go further than Bossé, by suggesting that part of the problem is that too many leading education bureaucrats and politicians believe that somehow

53. Both authors have served as consultants to this project.

54. The substance of Bossé's (1995) allegation has been contested by NCTM officials—see, for example, Burrill (1996).

curriculum developers, reformers, and policy makers have a right to exist as separate entities, set apart from teachers.

This has certainly been the case with moves to implement outcomes-based-education (OBE) policies in many nations (see, for example, Cuban, 1995; Ellerton & Clements, 1994; King, 1995; Noss, 1990). There are a number of countries where heavy-handed approaches have been used to thrust neo-behaviourist OBE policies on education systems and, ultimately, on unsuspecting teachers and students. Often huge funding has suddenly been found to support "needed changes" despite the fact that politicians and education authorities in central offices have been unable to find money for other professional development programs (see Marsh, 1994). Emotive terms like "accountability," "benchmarking" and "quality" have been used liberally by the advocates and by those likely to benefit most from the introduction of OBE approaches (Edsall, 1996; Zadkovich, 1996). Although, there was no adequate research base for OBE (Evans & King, 1994; Slavin, 1994), supporters were likely to be rewarded for following the party line, and opponents who dared to ask "difficult" questions were likely to be punished.

That is *not* the way to improve school mathematics. Furthermore, from an education research perspective, difficult ethical issues associated with how certain practices, and results obtained by education researchers, are perceived by bureaucrats and politicians have come to the forefront.

There is not only the danger of "friendly" researchers being "commissioned" to "evaluate" projects, but also the possibility that steps will be taken for "unhelpful" research findings to be suppressed, or for pressure to be placed on "hostile" researchers to refrain from speaking or writing about results that they have obtained. Indeed, there have been cases where, when research results have been reported or public statements made which were at odds with the party line, senior bureaucrats have written letters to employers of "offending" researchers, complaining about views publicly expressed by the researchers.

We have heard of instances in Australia, the United Kingdom, and the United States of America, where mathematics education researchers were told they would be denied funding for projects unless they were prepared to sign contracts giving the funding authorities the right to decide which results could be made public and which could not.

Mathematics education is obviously a politically sensitive matter, and mathematics education researchers are increasingly under pressure to toe the party line ... or else. This being the case, there is an urgent need for the international community of mathematics education researchers to develop a code of ethics. Ethical dilemmas[55] have been raised at a number of international gatherings of

55. For example, ethical dilemmas provided the focus for a "Hypotheticals" plenary session at the 1990 conference of the International Group for the Psychology of Mathematics Education, held in Mexico. The session was chaired by Alan Bishop, then of Cambridge University. Bishop, who is now at Monash University in Melbourne, read a paper (Bishop, 1996) on ethical considerations for mathematics education researchers to Working Group 24 at the ICME 8 Conference (held in Seville, Spain in July 1996).

mathematics education researchers, but firmer action is needed if mathematics education researchers are to retain their hard-won reputation for conducting independent investigations within a framework of scholarship and integrity.

The ethical dilemma needs to set against the different meanings which can now be legitimately associated with the term "mathematics education researcher." No longer should this term be applied only to some outside "expert" who enters an education environment, gathers and analyses data, and prepares a report on his/her findings. As Henriques (1996) stated, after describing the positive effects of a series of action-research projects which begun in 1992 and have now involved many teachers in the State of Madhya Pradesh in India:

> Not only do teachers own the project, but many attach tremendous significance to it as they perceive it as their only chance to improve their lot and status in society. To date, over 10 000 core group teachers have made tremendous sacrifices for the project and shown heroic commitment and dedication to their tasks at various levels like training, monitoring and providing support for their peers. (p. 360)

Henriques (1996) added that the success of the project has enabled Madhya Pradesh to make a huge leap towards achieving the goal of universal primary education within the State by the year 2000 AD. For their part, the teachers who are actively involved in the project have learnt to trust each other, have gained in status within their communities, and are "making their schools enjoyable for both teachers and the students" (p. 361).

It seems, then, that as we move towards the 21st century, the very meaning of "education research" has been rendered problematic. Mathematics educators are being called upon to work as equal partners within communities of reflective practitioners. At issue, too, is how universities will respond to this emerging situation. Will they, for example, follow the lead of the Aga Khan University, in Pakistan, and allow graduate students to work together on joint action research projects, often involving five or six researchers. If the answer is "Yes," then will the same universities allow joint research theses to be presented for assessment? Furthermore, it is interesting to consider who would be best qualified to assess such projects, and on what criteria they should be evaluated.

Bishop (1996), in elaborating a code of practice for mathematics education researchers, put forward a list of ten "maxims" which provide an appropriate conclusion to this chapter on action research:

1. Mathematics education researchers are part of the educational enterprise and as such have a primary responsibility to society as a whole ...

2. As mathematics education is a practitioner-dependent activity, the research process should therefore be practitioner-focussed ...

3. All research practices should be subject to strict ethical procedures ...

4. Institutional, social and cultural contexts should be recognised and documented in any research ...

5. Researchers in mathematics education should expose the assumptions underlying their research concerning "mathematics" and "education"...

6. Researchers doing experimental research should ensure that their experimental practices are not only ethically acceptable, but also potential beneficial to the participants ...

7. Any research in mathematics education should involve enquiry, evidence and theory, and should be justifiable in those terms ...

8. Researchers should be encouraged to criticise and challenge any research practices and procedures which do not in their opinion satisfy suitable moral and ethical standards ...

9. Researchers should endeavour to publish and disseminate their research conclusions in forms which are accessible to the maximum number of relevant practitioners ...

10. All research results and conclusions should be available for public and practitioner evaluation. (pp. 3–4)

With respect to his second maxim, Bishop (1996) commented: "This means that it [i.e., research] should address practitioners' issues and problems, and that practitioners should be involved at all stages of a research project, particularly at the start."

Mathematics education researchers[56] in the Asia-Pacific region could do no better than keep their eyes on Bishop's maxims as they design, conduct, and report investigations which seek to improve the teaching and learning of mathematics.

56. The term "mathematics education researcher" is being used inclusively, and includes practising teachers engaged in reflective investigations of their work.

6 Pencil-and-Paper Tests and Mathematics Education Research

Introduction: Historical Perspectives

This chapter provides brief commentary on how pencil-and-paper instruments came to be so widely used for the purpose of assessing mathematical learning. Several other contemporary issues associated with the assessment of mathematical learning are also raised. In addition, the question of when it is appropriate to use pencil-and-paper tests as instruments in mathematics education research is considered.

Any discussion of mathematics education research in the Asia-Pacific region should attempt to take account of the deep-seated acceptance by politicians, government bureaucrats and public servants, leaders of industry and commerce, and educators and parents, in the region, of externally-set, written "public" examinations as a legitimate and cost-effective sorting mechanism.

Indeed, the current emphasis, around the world, on the importance of written examinations for the purpose of achieving more accountable schools and education systems might be seen to have its origins in the long and widespread use of written examinations in Asian societies.

Examinations—The Soul of Asian Education

It is well known that public examinations in mathematics are not a new phenomenon. 2500 years ago, Confucius (551–479 BC) encouraged the systematic testing of students in family schools, village schools, city schools and at the national capital university (Cheng & Lu, 1993). Formal examinations, involving written answers to a series of prepared questions, were used by the Chinese in the 2nd century BC, and perhaps before then, to select recruits for the Chinese civil service. Those who were successful had to take the examinations afresh every nine years to retain their posts, and the highest official, the prime minister, was expected to prove that he was entitled to his superior rank by competing regularly and winning the highest place.

Since antiquity, then, the idea of candidates competing against each other in their efforts to win high positions in society, has been an inbuilt feature of Chinese

society. This attitude has spread across the Asia-Pacific region with the large-scale migration of Chinese peoples across and within the region and the willingness of many non-Chinese Asian cultures to take on the educational values of the Chinese.

According to Cheng (1995), public examinations were "a genius invention in ancient feudal China" and "have existed in China since the Sung dynasty for around a thousand years" (p. 8). Examinations were seen to be an efficient and equitable way of selecting public servants. Anyone could enter for the examinations, and success was a symbol of merit in that it was a reward for hard work, regardless of family background. As Cheng (1995) has pointed out, examinations came to be seen as a reliable means for facilitating and regulating social mobility:

> For centuries, public examination in China has given hope to millions of families for a better future of their younger generation. Although extremely few have fulfilled such a hope, public examinations have successfully pacified the underprivileged mass who would otherwise resort to revolutions. Public examinations have, on the other hand, also given education an extremely high status. (pp. 8–9)

Reliance on examination results, and indeed a strong faith in the fairness and efficiency of pencil-and-paper examinations, together with the Confucian heritage, have become part of the cultures of Japan (White, 1987), Korea (Robinson, 1994) and, according to Cheng (1995), in all communities which are predominantly Chinese.

It is not an exaggeration to say that, in the 1990s, public examinations are the soul of Asian education. Quoting once again from Cheng (1995):

> Examinations, university entrance examinations in particular, are important annual events in Japanese, Korean and almost all Chinese communities. Examinations have gone beyond their role of a selection mechanism; they are regarded as training opportunities for competition, adaptation, endurance, perseverance, and so forth. (p. 9)

It would be a mistake, however, to believe that public examinations are less important in Western societies than they are in predominantly Chinese communities.

Furthermore, from time to time important questions about the fairness of public examination systems have been asked by Asian politicians and educators. This has probably arisen because claims made that examinations provided equality of education opportunity did not seem to be true. For example, despite China's 2000-year tradition of civil service examinations, at the beginning of the twentieth century most Chinese had never attended school. For the children of the masses, the classical curriculum followed in schools was offputting. It is said, for example, that when Mao Zedong (1894–1986) entered a local grammar school in 1902 he was forced to recite "incomprehensible Confucian classics" (Karnow, 1972, p. 31), in preparation for the civil-service examinations. In fact, civil service examinations were abolished in 1905 (Hawkins, 1983), but formal national-level examinations remained an important component of Chinese education practice.

Mao Zedong often recalled his experience of examinations with bitterness. He resented, above all, the inability of a pencil-and-paper examination to get to the heart of what students can do:

> Examinations at present are like tackling enemies, not people. They are surprise attacks, full of catch questions. They are nothing but a method of testing official stereotyped writing. I disapprove of them and advocate wholesale transformation. (Quoted in Hawkins, 1983, p. 170)

In the early 1970s Mao Zedong's government attacked the traditional Chinese emphasis on national-level entrance examinations, charging that "the national entrance examination systems for enrolling college students was a major trick of bourgeois intellectuals to dominate the schools." It was claimed that although supporters of examinations declared that "everyone was equal before marks," the reality was that examinations "meant dictatorship of culture by the bourgeoisie aimed at shutting the door of colleges to workers, peasants, soldiers, and their children" (quoted in Hawkins, 1983, p. 171).

Since the late 1970s, however, China's new educational leaders have inaugurated a system in which there has been a dramatic return to the official acceptance of the worth of examinations—in formal, informal, and nonformal education settings. Hawkins (1983) stated that in his personal discussions with influential educators in China he found an eagerness "to learn of American testing procedures, even though educators in the United States seriously question the usefulness of many of the tests" (p. 184). Reporting on the results of the large-scale re-establishment of centralised examinations in China in the late 1970s and early 1980s, Hawkins wrote:

> If China is to avoid the problems that have resulted in many nations because of an uncritical total reliance on testing at all levels, then educational leaders must temper the obvious need to assess achievement and ability of students with a flexibility that will allow for alternative approaches. One of the major problems already emerging after a few years of administering the national college entrance examination is that of a skewed curriculum at the secondary level. It is reported widely that the middle-school curriculum is catering for preparing students for the college examinations while ignoring the needs of the vast majority of students who will never be admitted to higher education. Thus the curriculum has become lopsided, students are frustrated, teachers are ranked according to the number of their students who pass, and a major priority for students, parents and teachers alike is the acquisition of credentials. (p. 184)

Clearly, the fascination with testing which has characterised academic climate for centuries is alive and well. Even so, there has been some recognition of the disadvantages experienced by students from minority groups where the examination tradition is not well established. In some cases special examinations are administered to minority group children, and grades required to proceed to further education are lower than for mainstream students (Hawkins, 1983).

The main point to note here, however, is that the large role played by examinations in China's heritage, together with the strong vested interests supporting testing in all Western nations, continues to have an enormous influence on the thinking of politicians, education bureaucrats, teachers and parents throughout the Asia-Pacific region. Indeed, any mathematics education research carried out in the region which fails to take adequate account of this cultural heritage factor will be inadequate, and unhelpful.

Western Acceptance of Pencil-and-Paper Examinations: The Making of a Myth

One of the interesting features of Cubberley's (1948) 850-page treatise[57] on the history of Western education is the absence of any detailed description or analysis of the role of assessment in the education process. There is no entry for "Examinations" or "Tests" in the detailed Index to Cubberley's book.

It is not well known that pencil-and-paper examinations were not used in Western education until the nineteenth century. In the United Kingdom and in the United States it was not until the 1840s that examinations were introduced into schools and universities (Resnick, 1982). Prior to that, "public examinations" usually took the form of an "open day" at which parents and visitors were invited to be present and engage in public *disputation* with students (Kilpatrick, 1992, 1993a). This form of examination was also used in British colonies in the first half of the nineteenth century. The first matriculation examination at the University of Melbourne, for example, held in 1855, did not make use of pencil-and-paper tests (Clements, 1979).

However, once pencil-and-paper tests were introduced into European universities in the eighteenth century, possibly as a result of a felt need to emulate the objectivity of the Chinese civil service examinations,[58] and into civil service examinations in the nineteenth century, the idea rapidly transformed thinking and practice with respect to assessment in education and business sectors.

The November 1888 issue of the English periodical *Nineteenth Century* devoted 35 pages to a powerful "protest" against "the sacrifice of education to examination." Fourteen of the pages contained the names of some 400 well-known educationists, all of whom had agreed to attach their names to a statement which attacked "the dangerous mental pressure and misdirection of energies and aims" which were to be found "in schools of all grades and for all classes, and at the universities."

The *Nineteenth Century's* "protest" clearly represented a substantial attack on a phenomenon which, in a period of only fifty years, had grown to embrace the whole of the education scene in the Western world: this phenomenon was the externally-set, competitive, written examination based on externally prescribed courses of study. Forces of colonialism had meant that the use of the examinations had quickly spread to far-flung corners of the world. Thus, for example, in the West Indies in the second half of the nineteenth century, the Cambridge, Oxford and London University pencil-and-paper secondary school examinations were adopted, "without question as the graduating qualification in both boys' and girls' secondary schools." Indeed, examinations "conducted by the English examining boards became increasingly popular in the Caribbean" (Bacchus, 1995b, p. 60).

57. The first edition of Ellwood Cubberley's book *The History of Education* appeared in 1920.

58. And, possibly, too because of perceived weaknesses in the disputation system, for which it was alleged that many students did not engage in a search for truth, but rather in a display of individual power. In some schools, pupils were reported as wrangling, quarrelling, and sometimes fighting during disputations (Kilpatrick, 1993a).

Historical perspective makes it clear that the 1888 "protest" had no enduring impact. More than 40 years after the "protest," Alfred North Whitehead (1929/1962) was moved to write that "no system of external tests which aims primarily at examining individual scholars can result in anything but educational waste" (p. 25). However, such statements failed to galvanise opposition to a practice which responded so efficiently to a perceived need to monitor academic standards in the rapidly increasing numbers of schools.

Between 1931 and 1938 an "International Examinations Inquiry" occurred, and although many criticisms of objective testing were voiced at this Inquiry by eminent educators and psychologists, public faith in written examinations grew rather than declined. So far as the long-term effects of the Inquiry were concerned, Kilpatrick (1993a) stated that "there seems little doubt that it helped establish objectivity and reliability as desirable goals in an examination system by bringing forcefully to people's attention the degree to which uncontrolled marking schemes yielded highly divergent ratings of the same examinees" (p. 42).

After World War II the level of acceptance of pencil-and-paper tests for assessment of mathematics learning increased in many countries. This was partly related to the publication and influence of Benjamin Bloom's (1956) *Taxonomy of Educational Objectives*, which enabled items to be classified according to content and process. During the 1950s and 1960s many educators, and especially educators in Western countries, developed an implicit faith in the ability of pencil-and-paper tests and, in particular, tests of the multiple-choice variety, to generate quality measurements of education variables.

Large test construction bodies concentrated on the development of valid, reliable, and discriminating pencil-and-paper instruments. These bodies were often commissioned by governments to obtain "needed" educational measurements. An educational testing industry, nurtured and supported by public funds, was thereby developed. This industry had a vested interest in propagating the view that pencil-and-paper test instruments developed by test specialists (often called "psychometricians") were much more likely to provide objective, reliable, valid and cost-effective education measurements than any other form of assessment.

One such body was the Educational Testing Service (ETS), of New Jersey, whose advocacy of well-constructed multiple-choice tests knew no bounds. Consider for example, the following extracts from the ETS periodical *Multiple-choice Questions: A Close Look* (Educational Testing Service, 1963):

> The purpose of this close look at a group of multiple-choice questions is to dispel a myth: the myth of the multiple-choice question as a superficial exercise—one that requires little thought, less insight, and no understanding. Like other myths, this one may be based on a shadowy memory of the past, but it bears little relation to the present. ... Educational testing in its present form is a response to the needs of contemporary education. (p. 1)
>
> ... To create questions that are important, appropriate, accurate and discriminating requires the imaginative efforts of well-trained, experienced individuals working in concert. ... If a test is to provide reliable measurement, possible sources of error are best identified and eliminated in advance. Therefore, whenever feasible, questions are tried out experimentally before inclusion in a test. From such "pretesting" considerable information is derived.

> Each question in this booklet is followed by a statistical analysis that summarizes the kind of information obtained about the performance of those students tested with the question before its actual use in a test as well as after its actual use. (p. 3)

The ETS document may have referred to one myth, but it could be argued it was busy creating another—specifically, that pencil-and-paper tests, constructed by "expert" test organisations, and backed up by all manner of statistics (norms, reliability, difficulty, and item discrimination indices, etc.), are the best available instruments for measuring education variables. Governments accepted their claims—and so did most mathematics education researchers at that time.

Later in this chapter we shall summarise findings of some of our own research into the effectiveness of what are, ostensibly, "well-constructed" pencil-and-paper mathematics tests. Our data provide evidence that the education test industry developed, and maintained, a powerful but educationally destructive myth.

More recently, the Cockcroft Report, in the United Kingdom, stated:

> Examinations in mathematics which consist only of timed written papers cannot, by their nature, assess ability to undertake practical and investigational work or ability to carry out work of an extended nature. They cannot assess skills of mental computation or ability to discuss mathematics or, other than in very limited ways, qualities of perseverance and inventiveness. Work and qualities of this kind can only be assessed in the classroom and such assessment needs to be made over an extended period. (Cockcroft, 1982, Paragraph 532)

During the 1980s there was worldwide acceptance of the need to develop more effective assessment procedures for mathematics. Nevertheless, pencil-and-paper tests continued to dominate school mathematics (Garet & Mills, 1995), despite a recognition that they "are so reductionist to be misleading" (Holt, 1993, p. 169). Niss (1993a), in introducing a volume on "assessment" prepared under the auspices of the International Commission on Mathematical Instruction (ICMI), was moved to write that although the field of mathematics education had developed considerably in the area of ideals and goals, and theory and practice, "assessment concepts and practices have not developed so much" (p. 4).

It would hardly be an exaggeration to say that in the 1990s pencil-and-paper tests dominate school mathematics, despite the publication of numerous books and articles in which writers have sought to broaden the purposes, forms, and practices of assessment in mathematics education (see, for example, Clarke, 1992; Holt, 1993; Leder, 1992; Niss, 1993b,c; Webb & Coxford, 1993).

Equality of Educational Opportunity and Pencil-and-paper Tests for School Mathematics in the Asia-Pacific

The widescale acceptance of the importance of written examinations, together with the ever-increasing sophistication of communication technologies, has enabled competitive written examinations to remain a feature of many Asia-Pacific societies, despite the exponential growth of the numbers of students and teachers in the region. Equality of opportunity is provided for in the hypothetical sense that every person, independent of background, is able to rise to the highest possible levels within society by simply doing well on public examinations. According to

this line of thinking, the use of examinations means that appointments are more likely to be made according to ability and suitability, rather than through influence or nepotism (Yates, 1969).

Despite the acceptance within Asia-Pacific societies of written examinations, it needs to be recognised that recent mathematics education research is calling into question the power of externally-set written examinations to provide sound education accountability and equality of educational opportunity. Throughout the Asia-Pacific region—for example, in the Highlands of Papua New Guinea—there are, in fact, many cultural groups where the use of written tests is not, and never has been, part of the cultural heritage. The notion of competing for positions within a society is not something which all societies accept as desirable, and in any case, in most of these societies the idea that performance on a written test should form the basis of selection is preposterous.

Claims that equality of opportunity can be achieved through the use of written tests are not unambiguously supported by relevant data. Howson (1993) and Ridgway and Passey (1993) have provided useful outlines of some of these data from various parts of the world, and the evidence does not appear to be good for those who argue that examinations have a role to play in bringing about social change. For example, from a list comprising 1627 names of successful candidates for the Korean mathematics examination over the 400-year period 1400–1800, it was found that the occupations of the fathers of these candidates were as follows: 124 herbalists; 75 translators; 6 astronomers; and 1422 mathematicians (Howson, 1993).

The fact that a society has great faith in the power of an objective and reliable public examination system to generate a well-ordered and stratified community does not mean that that faith should not be the subject of rational analysis. One can ask a question such as: "Is a test *really* fair if some of those asked to do it cannot demonstrate relevant understandings because they cannot read or comprehend the written questions?" This is an important question in the Asia-Pacific region, which contains over half the world's languages, some of which until recently have been in oral forms only (Lean, 1992, 1995). Indeed, millions of children in the region are expected to learn mathematics in classrooms where their first language is not spoken. In many of these classrooms, the language of instruction is, in fact, the third or fourth language for most students (Clements & Jones, 1983; Secada, 1988). If written tests are not in the students' first language, then how can these instruments be the basis for achieving equity and justice?

Even within nations, such as China, where externally-set written examinations have been widely used for many years, it can be argued that such examinations favour a particular stratum of society where there is minimum "distance" between the discourse forms represented in the examination paper, and in the personal worlds of those taking the examination. The likelihood that the great majority of Chinese people accept the role of externally-set, written public examinations in their society (Cheng, 1995) does not diminish the probability that such examinations serve to perpetrate injustices.

We know that in Australia, for example, the creation of a mass system of high schools after World War II resulted in middle and upper status groups resorting to tus reproduction strategies such as the use of exclusive and expensive private ools and private tuition services to "buy" good examination results (Teese,

1995). It is interesting to note that Jagan (1979) made a similar point in his analysis of matriculation results of students in secondary schools in Melbourne over the period 1858–1880. Howson (1993) has drawn attention to the same phenomenon in the United Kingdom. Despite attempts, for over 150 years, to use examinations to establish a more meritocratic society, Cambridge and Oxford Universities still receive more students from non-government schools than from government schools, even though there are far more students overall in the latter.

The concept of equality of educational opportunity must therefore be recognised as extremely difficult to define (Apple, 1995), and very difficult to achieve through examinations. In fact, regardless of any agreed-upon definition, it could be argued that in any society where there is an official determination to use externally-set written examinations as major selection devices, it will be impossible to achieve equality of educational opportunity (Bishop, 1993). Such a possibility should be taken seriously by every person reading this book.

Cuban (1995) has pointed out that although California, Texas, and New York introduced, in the 1980s, statewide content standards and curriculum-based tests, their big-city schools continued to have sorry records. They continued to "struggle with high drop-out rates, spotty attendance, and dismal academic performance, especially is secondary schools ... which seemed resistant to higher graduation requirements, state curricular frameworks, incentives and sanctions, and publication of school-by-school test scores in local newspapers" (p. 52). King (1995), an expert on Black education studies, has argued that the imposition of state or national tests will generate more, rather than less, equality of educational opportunity. She stated, categorically, that bottom-up reform strategies are to be preferred to "the imposition of national curriculum standards and high-stakes tests" (p. 136).

In the United Kingdom, a poll of teachers in 1993 found that 98% of the teachers felt that planning and preparation for the national pencil-and-paper tests had been inadequate and the demands of the statewide assessment were so great they could not devote sufficient attention to normal teaching. In fact, 91% of teachers were so upset that they were prepared to boycott the assessment tasks—a threat which was implemented when the National Union of Teachers voted overwhelmingly (96%) not to administer the national tests to 14-year-olds (Kellaghan & Madaus, 1995).

What we are saying is that most of the assumptions concerning externally-set, written examinations do not stand close scrutiny. And we are not the only ones saying this. Volmink (1994), writing with passion from a South African perspective, has stated categorically that tests based on atomised approaches to mathematics curriculum, such as is the case with short-answer and multiple-choice written tests, have had a "pernicious effect on mathematics education because they have tended to devalue, dismiss, and separate people" (p. 63). Volmink went on to assert that those who are serious about the struggle for social justice and equality should actively pursue ways by which mathematics learning can be assessed in non-discriminatory ways. Assessment should "celebrate the value of each person" and "be illuminatory instead of discriminatory; it should reveal value rather than merely identify deficiency" (p. 63).

That Volmink's (1994) argument is something more than mere rhetoric could be [illustrated] by examples of the unfortunate effects of examinations on students in

any nation in the Asia-Pacific region. We have chosen one such case. Early in 1996 a 15-year-old boy who had been attending an élite university demonstration secondary school in Thailand came under such pressure from the competitive examination system that he committed suicide. This precipitated a discussion in Thai newspapers about whether the emphasis on examinations, and on science and technology, in Thai secondary schools was serving the nation well. The following is an excerpt on the issue from an editorial which appeared in *The Nation*, a Thai English-language newspaper, on February 15, 1996:

> Polarization in such schools [i.e., élite secondary schools] is becoming deeper year by year as Thai society becomes more and more commercialised. Demonstration school students consider themselves élitist, and will only mix among colleagues whose parents are in the same social or financial status. The school authorities have to be blamed for this bearing in mind that a lot of attention is normally showered on students who rank among the top 10 in the class, while those in the bottom rung are often ignored.
>
> Of course rich parents can afford the best for their children. Extra private tutorial classes at exorbitant rates mean nothing to them, while middle class and poor parents trying to make ends meet will have to scrape the bottom of the barrel if their children are to have this luxury.
>
> The rigid Thai educational system that places so much emphasis on public examinations fails to take into account that a rich student normally has the edge over a middle class or a poor one. (p. 4)

It would be impossible to assess the extent of the psychological damage to individuals, and the collective damage to society, brought about by competitive pencil-and-paper public examinations. Yet, politicians, bureaucrats, and indeed people from all walks of life, seem to believe that a strong competitive examination is the cornerstone of economic progress and equality of educational opportunity.

Bishop (1993), who has also argued against the overwhelming desire of leaders in societies to use mathematics examinations as a means by which élites will reproduce themselves, provided the following translation of a powerful quotation from Revuz, a French educator:

> The quite proper demand for the wide dissemination of a fundamental mathematical culture has boomeranged and has been transformed into a multiplicity of mathematical hurdles at the point of entry into various professions and training courses. A student teacher is regarded as of more or less value in accordance with his level of mathematics. Subject choices which make good sense in terms of their relevance to various jobs have been reduced to the level of being "suitable only for pupils who are not strong enough in mathematics; instead of attracting pupils by their interest, they collect only those who could not follow, or who were not thought capable of following, a richer programme in mathematics. Success in mathematics has become the quasi-unique criterion for career choices and the selection of pupils. Apologists for the spread of mathematical culture have no cause to rejoice at this; this unhealthy prestige has effects diametrically opposed to those they wish for.
>
> The cause of this phenomenon is undoubtedly the need of our society, as at present constituted, to try to reproduce a selective system which runs counter to attempts at democratisation which are going on simultaneously.

> Latin having surrendered its role as a social discriminator, mathematics was summoned involuntarily to assume it. It allowed objective assessment to be made of pupils' ability, and it seemed amidst the upheaval of secondary education in general, to be one of the subjects which knew how to reform itself, which stood firm and whose usefulness (ill understood, truth to tell) was widely proclaimed. (Translated by D. Quadling from Revuz (1978, p. 177), and quoted in Bishop, 1993).

This chapter therefore raises fundamental questions which challenge important aspects of the education cultures which have been established and nurtured throughout the region.

Reactions Against Competitive Written Examinations

Although during the twentieth century the seriousness of problems associated with using competitive written examinations to create meritocratic ladders based on performance was recognised in most countries, it was only occasionally that politicians and educators dared to take decisive steps to change things. One notable occasion was in 1918, when Russia removed all university entrance requirements. Examinations and gradings were abolished on the grounds that they led to formalism in teaching and discouraged creativity among pupils. Another was when Mao Zedong reformed the Chinese education system, proclaiming that in the past in China "teaching had been by injection instead of through imagination," and examinations had been used to ambush well-meaning students (Howson, 1993).

One of the most interesting cases occurred in South Korea in the 1960s, following a sharp increase in the number of pupils seeking entrance to the nation's best middle schools. Competition for places in the schools had become so fierce that the expression "entrance-examination hell" was coined.[59] Curricula were distorted, topics treated in an atomistic way, and primary schools became places characterised by rote teaching and learning. Wealthy, and not-so-wealthy parents of primary school children employed private tutors to prepare their offspring for the forthcoming middle-school entrance examination (Hong, 1983).

In 1968 the Ministry of Education in South Korea decided to act decisively on the matter. It abolished the middle-school entrance examination, substituting in its place a lottery system combined with an entrance-according-to-home-location system. According to Hong (1983), this decision immediately had an impact on education culture in Korea. Private tutoring for primary school children was eliminated; less emphasis was given to some subjects and more to others in primary classrooms; much less undue emotional pressure was placed on children and on families; middle-schools came to be regarded as more or less equal in efficiency; and society began to wonder whether entrance examinations should also be abolished for higher levels of schooling (Hong, 1983).

In 1973 a further attempt was made to alleviate "exam-hell" by introducing a "high school equalisation policy" by which the number of places in high schools was increased and applicants were randomly assigned to high schools within the cluster of academic high schools. The Ministry of Education also standardised entry to college by instituting a national, multiple-choice examination which,

59. According to Woo (1993), "there are two terrible hells in Korea: one is the college entrance exam-hell, and the other is the traffic hell in Seoul" (p. 153).

according to Park (1993), did not satisfactorily evaluate students' performance, in the sense that the type of examination was inconsistent with course objectives. The national entrance examination "had a great influence on the type of mathematics instruction in high schools and application, and creativity in math education was ignored" (Park, 1993, p. 143).

In fact, according to Woo (1993), recent statistics show that over 93% of Korean parents expect their children to enter college or universities. Examination-oriented education is deeply rooted in Korean values and cultures, originating from a 10th century state examination system called "Gwageo Jedo." Education is viewed as the major vehicle for upward social mobility, and the result, according to Woo (1993), is that schools have degenerated "into institutions for preparation for college entrance examination"—instead of being places where more noble goals of education are pursued. "Private tutoring thrives in an effort to give an added assurance of success in the competitive bidding for entrance examination," and "students are physically and mentally over-strained by rigorous preparation" to the extent that they have become "slaves of examinations" (Woo, 1993, pp. 152-153).

Hong (1983), saw the overall effect more positively:

> Since the mid-1960s educational planners have altered the entrance-examination system at all levels of secondary and higher education in order to reduce the emotional strain imposed on young adolescents, to encourage more capable students to enter vocational high schools, and to control the spread of private colleges that admit unqualified high-school graduates. (p. 231)

It would not be appropriate to trace these developments further, here, except to reiterate Howson's (1993) comment that in all cases where major change was attempted "examinations soon made a comeback" (p. 51). The main point to note here is that changing forms of assessment can have important effects on learners' views of the nature of mathematical thinking and on what transpires in mathematics classrooms. The history and influence of theories and practices relating to the assessment of school mathematics should provide *all* mathematics education researchers with much food for thought.

Two Examples of Learners for Whom Capital M Mathematical Knowledge Was Useless—Except, Perhaps, for the Purpose of Passing Examinations

One of the main reasons why many mathematics education researchers are turning towards anthropological methods of investigation is that they now realise that so much of what has become acceptable practice and behaviour in standard mathematics lessons in many countries[60] does not help children to understand mathematics. Indeed, the sub-cultures associated with many school mathematics programs have been destructive in the sense that they have given many students a false sense of security—the students have been persuaded that they have understood something when, in fact, they have not. Anthropological investigations can lay bare the unsatisfactory aspects of what goes on in mathematics classrooms,

60. Though not necessarily all countries—see Stigler and Baranes (1988).

and can provide data which will assist mathematics educators to define what constitutes quality mathematics learning environments.

We now provide two classroom examples which illustrate how a narrow emphasis on drill and practice, on getting right answers, can in fact be educationally useless. In both examples the teachers were capable, sincere and dedicated persons with the best interests of their students at heart. In fact most experienced observers would have regarded them as "good teachers." The reader is asked to reflect on whether the type of outcome which is revealed is typical of what is achieved in most mathematics lessons conducted around the world.

Example 1: Multiplying with fractions. Clements and Lean (1988) reported research in which a whole class of Grade 5 children had been drilled on how to multiply fractions. Before Clements and Lean interviewed the children the class had devoted considerable time to finding answers to questions like: "Find the value of 7/11 x 792." The children tackled such tasks in a mechanical way: for example, in the particular case mentioned they tended to divide 11 into 792, "cancel" the 792 and put 72 in its place (if they got the division correct), multiply 72 by 7 and, if they got the multiplication correct, write down 504 as their answer. They checked their answers at the back of the book, put ticks or crosses depending on whether they were right or wrong, and then went on with the next multiplication involving fractions.

When the children were interviewed it became clear that not one child in the class had any idea of what 7/11 might mean. They did not know why they had divided 11 into 792, except that this was what the teacher had told them to do, and this was how they "got the right answer." No child was able to make up an example in real life where someone might want to find out the value of 7/11 x 792. When shown 12 stones, and asked to give 1/4 of them to the interviewer, quite a few of the students who had got 7/11 x 792 correct gave the interviewer 4 stones!

Example 2: The area of a triangle. One of the authors (Clements) watched a Grade 8 lesson, in an Australian classroom, on "the area of a triangle." The lesson began with the teacher reminding the students that in the previous lesson they had learned how to find the area of a rectangle. When the teacher then asked the revision question, "How do you get the area of a rectangle?" no one volunteered an answer. On asking a particular student for the answer, the student, looking somewhat embarrassed, stammered, "180?"

Ten minutes later, after the teacher had shown the class how any triangular region could be regarded as half a rectangular region, the formula $A = (b \times h)/2$ was written on the board and then a worked example of how this could be used was provided by the teacher. The students were then told to turn to a page in their textbook on which there were 20 exercises which had exactly the same structure as the worked example (the length measures of the base and the altitude changing for each exercise). The students quickly tackled the exercises, and seemed to be pleased that they were getting correct answers. But when some of the children were subsequently interviewed it became clear that they could not distinguish between "area" and "perimeter," and that they *thought* that in calculating the areas they had found the distances "around the outside" of the triangles. In fact, these children had merely followed a rule which enabled them to obtain correct answers (Clements, 1995b).

Questioning the Role and Format of Examinations

Written Examinations: Pragmatic, Cost-effective, and Accepted by Society?

It is often argued that, in particular, 'the use of short-answer formats or multiple-choice formats necessarily precludes the assessment of higher-order thinking and mastery of complex material (Holt, 1993). Despite sentiments such as these, some influential teachers, education administrators, psychometricians, bureaucrats, politicians, and international aid organisations such as the World Bank advocate the continued use of such tests on both educational and pragmatic grounds. These advocates claim that well constructed short-answer and multiple-choice tests are not only cost efficient but are also reliable and valid from a psychometric perspective (Lockheed, 1996).

Indeed, bureaucracies increasingly claim that externally-set short-answer or multiple-choice tests—especially (but not only) in literacy and numeracy—can provide teachers, schools and school systems with quality data which would not be available if the schools were left to themselves to assess, monitor, and report on the progress of their own students (Holt, 1993). Pencil-and-paper tests are therefore being seen as key instruments for educational accountability.

Garet and Mills (1995) provided data on changes in school mathematics in the United States of America over the past decade which showed that there had been no reduction in the use of traditional pencil-and-paper short-answer and multiple-choice tests. They claimed that although there many positive changes in school mathematics programs had been effected by the introduction of the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989), pencil-and-paper tests, mainly of the short-answer and multiple-choice varieties, still constituted the dominant form of assessment.

This move towards accountability led Shavelson, Baxter, and Pine (1992) to comment:

> In the final analysis, we suspect that this nation [that is to say, the United States of America] may be placing far too much weight on accountability to achieve its reform agenda. Judging from experience, those states with the strongest and most technically sound accountability systems have not achieved their desired reforms ... Perhaps what is needed is far less account taking and far greater consideration and resources given to teaching and learning, especially for students drawn from diverse social, economic, cultural and language backgrounds.
>
> An increased emphasis on and bully-pulpit use of highstakes testing may, paradoxically, have a deleterious effect on U.S. education. Tactical, political "fixes" are not what is needed. Rather, we believe that a long-term, realistic approach to assessment—one that transcends politicians' terms of office—is. (pp. 26–27)

Political realities render it almost certain that pencil-and-paper short-answer and multiple-choice tests will continue to be widely used across the world for assessing the mathematics performance of school children.

In Australia, for example, schools, short-answer and multiple-choice pencil-and-paper tests are still widely used for the purpose of assessing mathematical understanding (Olssen, 1992). However, there is a rapidly

developing degree of cynicism with respect to arguments for accountability by politicians, representatives of testing agencies, bureaucrats and politicians. Edsall (1996), for example, has described accountability in education as being "answerable to the rules and guidelines of the corporate sector," the assumption being that teachers "are not accountable enough" and that if they were brought before the "Court of Accountability" they would be "found guilty, sent to dungeons, and replaced by better (cheaper, non-unionised) replacement workers" (p. 16).

These kinds of sentiments reveal the reaction during the 1990s of teachers in all Australian states and territories, to government-controlled external testing regimes which have become an integral part of school mathematics cultures. It would not be an exaggeration to say that most education administrators, and politicians believe that pencil-and-paper tests, and especially those of the short-answer and/ or multiple-choice variety, provide the most cost-effective means of monitoring and assessing mathematical achievement in our schools. These tests are key instruments for checking on whether value-added learning has occurred with respect to benchmarks and outcome statements. According to Edsall (1996), however, Australian teachers are increasingly regarding outcomes-based -education (OBE) as "an attempt to de-intellectualise teaching, to appropriate the teacher's voice from the classroom, and in the guise of teaching students instead of subjects, to rob students of their democratic right to discuss and discover a broad range of issues" (pp. 16–17).

In the United Kingdom the national testing of students at different year levels is through pencil-and-paper tests. The largest mathematics competition in the world, the Australian Mathematics Competition, is based on results from pencil-and-paper tests, and the International Association for the Evaluation of Educational Achievement (IEA), which is supported by funds provided by governments around the world, has used pencil-and-paper tests in its three major international mathematics achievement studies.

Summary of Research into the Effects of Standardised Testing

Despite the widescale, and increasing use of statewide and national pencil-and-paper testing, one rarely finds succinct and reasonably objective commentaries on the effects of such forms of evaluation on students, teachers and schools. A useful summary was provided, however, by Kindsvatter, Wilen and Ishler (1988), who drew attention to a substantial body of literature which describes the effects of large-scale, standardised testing for accountability. Data indicate that the use of such tests does not raise standards, and teachers consider that standardised tests do not assist learning. In fact, there is no conclusive evidence to show that assessment of student learning from externally-set pencil-and-paper tests provides better quality data than could be provided by teachers' ratings of students.

Standardised tests clearly have their greatest influence outside the classroom, and are primarily used for established benchmarks from which the performances of schools and systems of schools can be monitored and compared.

It is not widely known, however, that research has generated data which

suggest that despite enthusiastic claims to the contrary by psychometricians and education bureaucrats, performance data generated by externally constructed, pencil-and-paper tests can be seriously misleading. There is, in fact, increasing evidence pointing to the conclusion that students who answer pencil-and-paper mathematics items correctly sometimes have little or no understanding of the mathematical concepts and relationships which the items were designed to measure, and that this applies even for so-called "valid" and "reliable" tests (Frary, 1985; Gays & Thomas, 1993; Hembree, 1987; Thongtawat, 1992).

Despite this questioning of the effectiveness of pencil-and-paper tests, education authorities continue to believe that externally-set pencil-and-paper tests can satisfactory measure student understanding of mathematical knowledge, concepts, skills, and principles. A large research study into this question, jointly supervised by the authors and carried out in Thailand by Thongtawat (1992), found that the proportion of students giving correct answers to multiple-choice mathematics items but who did not understand the mathematical concepts and relationships involved in the items, was much higher than that for corresponding short-answer but not multiple-choice items. Thongtawat also found that students who scored poorly on a test could sometimes have a good conceptual grasp of the material which the items covered.

The authors of this book have also carried out research designed to test the effectiveness of multiple-choice and short-answer pencil-and-paper tests (Ellerton & Clements, 1995). Interestingly, in a study in which 116 Year 8 students in five secondary schools in two Australian states were interviewed, we found that both multiple-choice and short-answer questions were seriously ineffective in their assessment of student understanding. In fact, over one-third of responses to well constructed questions in multiple-choice and short-answer formats were such that either (a) correct answers were given by students who did not have a sound understanding of the correct mathematical knowledge, skills, concepts and relationships which the questions were intended to cover; or (b) incorrect answers were given by students who had partial or full understanding. Almost 50% of the incorrect responses were given by students who *did* understand, at least partially, the mathematics that the questions were designed to assess (Ellerton & Clements, 1995).

We concluded the discussion of our research findings with the following statement:

> We are particularly concerned about the increasingly widespread use of externally-set, multiple-choice and short-answer pencil-and-paper instruments. Testing regimes, based on crude, one-off, pencil-and-paper instruments are employed, and justified by reference to the need to make teachers, schools and systems more "accountable." The data in this paper suggest that claims that such tests can provide "quality assessment data" about students' and schools' mathematical performances are not justified. (pp. 12–13)

In the same paper, we questioned whether multiple-choice questions could ever be valid for assessing mathematical learning, pointing out that we do not know of any practising adult mathematicians who actually work in situations where they are regularly asked to choose one correct answer from four or five possibilities. The

implication is that schools which use multiple-choice tests as a major method for assessing mathematical learning are in danger of continually reinforcing in their students' minds a faulty image of the nature of mathematics.

The findings of our research fly in the face of continued widespread usage of pencil-and-paper tests to assess mathematical learning in all parts of the world. Educational testing is a multi-million dollar industry in most countries, and there are strong vested interests which are propogating the view that pencil-and-paper mathematics tests provide the most objective and cost-efficient means of measuring mathematical learning. Arguments calling for benchmarking the mathematical performance of schools and school systems, through "value-added" procedures, are being put forward by testing agencies and education bureaucrats (Cuttance, 1995; McGaw, 1995).

In Australia, full cohort "high stakes" statewide testing is commonplace. Yet, our evidence would suggest that if pencil-and-paper mathematics tests are being used (as is indeed the case) then it is inevitable that invalid results will be obtained. Furthermore, this testing occurs despite well-known reasons why high stakes tests—that is to say, tests designed for selecting students for advanced education, training or employment opportunities—have adversely affected education practice. Shepard (1991) pointed to six such reasons:

1. Sequences of high stakes test scores can produce higher scores without actual improvement in learning taken place—in that sense, they are unsuitable for benchmarking.

2. The use of high stakes tests can encourage school administrators to narrow their curricula, in order that students will devote more time to what is likely to be tested.

3. High stakes testing can influence teachers to teach to the test; this can lead to mechanistic instruction, especially for the "basic skills."

4. Items on high stakes tests often deny students the chance to demonstrate their higher-order thinking and problem-solving capacities.

5. It is easy for school administrators who are overseeing schools where high stakes tests are employed to find reasons why hard-to-teach children should not be in their schools.

6. The use of high stakes testing can demean the role of teachers, and ultimately reduce the status of the profession.

Cuban (1995) expanded on these and other consequences of high stakes pencil-and-paper testing programs in schools in the United States. In sections, entitled "Unintended consequences for schools" and "Indirect but unintended consequences" (pp. 52–58), he lamented the "well-intentioned but ultimately misguided experiment of a national curriculum and tests visited upon millions of classrooms" (p. 59). Education bureaucrats and school administrators in the Asia-Pacific, who have a history of rushing to mimic patterns of education practice and evaluation which have been put in place in the United States, should pay careful attention to these warnings of the unintended, but nonetheless damaging, effects of large-scale testing programs.

Assessment, Pencil-and-Paper Tests and Mathematics Education Research

Critiquing Classical Test Construction

Given that recent research has suggested that an educationally significant proportion of written responses to pencil-and-paper multiple-choice and short-answer mathematics questions do not, by themselves, provide an accurate indication of the level of the respondents' mathematical understanding (Ellerton & Clements, 1995), the question arises whether it is still legitimate to use such instruments in mathematics education research. If it *is* legitimate, then *how* should such instruments be used?

Elsewhere, we have, in fact, claimed that so-called "valid" and "reliable" pencil-and-paper instruments can provide seriously inadequate measures of mathematical understanding (Ellerton & Clements, 1995). In particular, formula-driven statistical measures of test reliability (for example, the Kuder-Richardson 20 reliability index) can be misleading in the sense that a mathematics test deemed to have high content validity by experienced teachers and a high reliability may not generate measures of mathematical understanding which are validated when students' levels of understanding are investigated by other means (like, for example, Newman interviews).

In fact, the widely accepted view that the classical test constructs of test reliability and test validity provide adequate criteria for the development of sound research instruments ought to be called into question. This raises special problems for mathematics education researchers who are striving to select and/or develop "valid and reliable" instruments for their research.

Pencil-and-paper Tests and the Operationalising of Variables in Mathematics Education Research

In the 1990s, research into why elementary and junior secondary school students make mistakes on arithmetic word problems on pencil-and-paper tests has consistently indicated that well over half of the errors are associated with difficulties with what Newman (1977a,b) called Reading, Comprehension, and Transformation (see Bulbert, 1995; Ellerton & Clements, 1992b). Analysis of written responses, without any other form of analysis, does not reveal the proportion of correct answers given by students who have little or no understanding of the mathematics involved, or the proportion of incorrect answers which were given by students with good understanding.

Given that recent research suggests that up to one-third of all responses are in one of those categories (Bulbert, 1995; Ellerton & Clements, 1995; Forbes, 1995; Graham, 1995), it appears to be the case that mathematics education research which operationalises and measures a dependent or independent variable through a single application of a pencil-and-paper research instrument is on very shaky ground indeed. Since numerous research studies, conducted in many countries, have relied heavily on such an approach, a review of what constitutes "good

design" for quantitative studies in mathematics education research is urgently needed.

An example of an evaluation study which failed to capture the essence of numeracy. Although many examples of education research studies in which dubious inferences have been made from data based on a single pencil-and-paper mathematics instrument could be cited, it will be sufficient—for the sake of argument—to discuss a fairly typical case.

Mappu (1982) carried out a study in Indonesia comparing the effectiveness of a "booklets only" distance course with a "booklets + radio broadcasts" course. The numeracy program was part of a larger program which was designed to assist about 24000 Indonesians aged 15 years or more to acquire basic literacy, numeracy, and general knowledge skills. Mappu used a complex, multi-stage sampling procedure to obtain two representative samples containing, altogether, 600 students—300 in the "booklets only" course and 300 in the "booklets + radio broadcasts" course. According to Mappu (1982), his sampling procedure "ensured that within the experimental target population and within the control target population, each student had the same chance of selection," and consequently, "within the experimental sample and within the control sample it was possible to conduct unweighted data analysis" (p. 33).

Given the care with which he obtained his samples, it was to be expected that Mappu would have used satisfactory instruments for the purpose of obtaining post-course measures of literacy, numeracy, and general knowledge performance. In fact he used classical test development procedures to develop three multiple-choice pencil-and-paper instruments—one for each of literacy, numeracy and general knowledge.

According to Mappu (1982), the items on the tests were based on a content analysis of the booklets which were used by all students. For each instrument a grid of objectives based on the content of the booklets was prepared, and this content analysis was checked by a separate team. For each objective, a series of items was written, reviewed and rewritten. The items were then trialled and classical item analysis carried out. Items with poor discrimination were dropped, and where necessary better distracters were introduced, and some item stems were rewritten. The validities of the final tests were judged by cross-checking the tests against the weight accorded to each objective in the courses and the relative weights in the tests.

The final form of Mappu's (1982) numeracy test had 20 items: 3 covered addition and subtraction, 7 multiplication, 6 division, and 4 percentages. Test reliability was a modest 0.58, and the highest score obtained by any of the 600 students was 15. Because of the tight procedures used in the test development process, the test was deemed to be content valid.

The average scores (out of a possible 20) of the two groups of 300 students on the numeracy test were 4.7 for the "booklets only" group, and 4.4 for the "booklets + radio broadcasts" group. Since each of the 20 items had four possible answers, one would expect random guessing to have produced a mean score of 5.0. On this basis Mappu (1982) concluded that "neither group was numerate", and that it appeared to be the case that "no numeracy had been learned at all" (p. 43). This

disappointed Mappu, because he had expected that both groups would have developed numeracy skills and that the "booklets + radio broadcast" group would have done better than the "booklets only" group. Further analysis revealed that, if the criterion was performance on the numeracy test, then none of the numeracy objectives had been met by either group, with less than one-quarter of the students having given correct answers to questions involving multiplication, division and percentages.

Mappu (1982) concluded that the numeracy program, either by booklet only or by booklet and radio, had not worked, and that "something would appear to be seriously wrong" (p. 45). According to Mappu there could have been many reasons for the numeracy programs being unsuccessful, including poor initial sequencing of materials in the booklets and the radio broadcasts, "or lack of relevance to the daily lives of the participants" (p. 50). He suggested that site visits should be made to attempt to discover the reasons. Following these visits, any necessary revisions would be made to the texts, the revised texts would be evaluated in the field, and final revisions made.

It seems to us that Mappu's criterion-referenced definition of numeracy was far too shallow, and that his pencil-and-paper, multiple-choice numeracy instrument failed to capture the essence of what numeracy meant for people aged 15 years or more who were living in remote parts of Indonesia. Indeed, the study would appear to provide a good example of how an *apparently* well-constructed multiple-choice pencil-and-paper numeracy test, which was deemed to be content valid and was trialled and improved before its use in the main study, was totally inadequate and inappropriate for the purposes of measuring what it claimed to measure. In the absence of observational and interview data we believe it was presumptuous to conclude that "neither group was numerate." There would appear to be a strong case to support the view that, in fact, the research design was fundamentally flawed because numeracy of the kind that Mappu (1982) intended to measure cannot be adequately measured by a multiple-choice, pencil-and-paper test.

We would wish to add that we believe that all the sophisticated multivariate analyses in the world will not be helpful in addressing education research questions if instrumentation for key variables is inadequate.

Biasing a research study by using pencil-and-paper instruments. The emphasis in schools and other education institutions on achievement on pencil-and-paper tests of mathematics has often made it difficult for educators to recognise that research designs in which pencil-and-paper tests are used to measure key mathematics variables can bias the research towards the likelihood of gaining results which support traditional teaching methods. The Indonesian evaluation study discussed above is, probably, one such study. Another study of the kind was one carried out in Liaoning Province, China, by Wei and Zhang (1993), in which the aim of an intervention program was to improve students' performance on pencil-and-paper mathematics tests. When this was achieved, the program was deemed to have been successful.

In studies such as these, quite different results might have been obtained if the test measuring the dependent variable had not been of the pencil-and-paper riety, but had involved people in real-life interchanges in which mathematical ills were involved.

In much of the research into what constitutes effective mathematics teaching (see, for example, Bourke, 1984), the criterion tests for measuring the effects of teaching have almost invariably been performances on tests of the pencil-and-paper variety— often in multiple-choice format. It could be argued that this makes it more likely that this research is biased in favour of those types of teaching which have been designed around achieving good results on pencil-and-paper tests. In the following passage, from Good and Biddle (1988), the emphasis on relating teaching to achievement gains should be noted:

> It is instructive to note that in our experimental work active teaching was an important difference between teachers who were getting good achievement gains and those who were getting lower-than-expected gains. Teachers whose students made higher gains were much more active in presenting concepts, explaining the meaning of those concepts, providing appropriate practice activities, and monitoring those activities prior to assigning homework. The fact that these teachers appeared to look for ways to check whether their presentations had been comprehended by students was particularly important ... Active instructional efforts seem to be an important aspect of teaching that is related to achievement gain, at least in basic skill areas. (p. 130)

On reading passages such as this, a cynical commentator might say that teachers "who teach to the test" are likely to be seen as effective.

Grouws and Meier (1992) have acknowledged that the reliance on standardised testing was a weakness of effective-teaching research. Although the tests did focus on the kinds of "basic skills" about which society expresses so much concern, "still, this narrow assessment tended to trivialise the scope and nature of learning outcomes that teachers should be expected to foster, and many student outcomes went unmeasured" (p. 88). According to Grouws and Meier (1992), "higher-order thinking skills, understandings, and conceptualisations, for example, were generally ignored, as were affective outcomes" (p. 88).

Similarly, research into the possibilities and effectiveness of various approaches and technology (like, for example, the effects of hand-held calculators on the numerical skills and understandings of primary school children) has tended to use pencil-and-paper tests to operationalise the dependent variable (Penglase & Arnold, 1996). Thus, for example, when Cheng and Lu (1993) investigated the effects of a middle-school mathematics program in which a wide range of enlightened assessment procedures were used—including the monitoring of students' performances and attitudes by diagnostic, formative, and summative instruments—the criterion for deciding whether the program had been successful was student performance on the Hubei Province pencil-and-paper, middle-school mathematics test.

One must ask: How valid is it to use a pencil-and-paper test as a dependent performance variable in such research? What are the implications of the fact that at the primary school level relatively inexpensive calculators are now readily available to pupils? Should not contemporary curricula be different from curricula in previous eras when calculators were not readily available and there was a much greater emphasis on pencil-and-paper algorithms? Similar curriculum and assessment questions arise for the middle and senior schools, as a result of the availability of contemporary hardware and software (like, for example, CD-ROM,

and calculators, LOGO, *Cabri Geometry*, and advanced mathematics software such as *Derive* and *Mathematica*).

On the matter of research into the effectiveness of graphics calculators, Penglase and Arnold (1996) have asked:

> What kinds of achievement instruments should be used in research which seeks to compare the effectiveness of two or more methods or teaching approaches, or classes where graphics calculators are used and classes where they are not used? If these instruments were developed prior to the era of graphics calculators, or even just prior to the introduction of graphics calculators into a course of study which is used as a foundation for research, then they could very well be biased in favour of traditional methods and approaches. For example, when working with the graphics calculator, considerable attention needs to be given to working out appropriate scales on x- and y-axes. Also, there is a significant emphasis upon the connections between symbolic and graphical representations as well as the inspection of graphical solutions by "zooming" in. Are these skills reflected in the tests used? (p. 31)

Such a statement raises the issue of how researchers ought to measure the effectiveness of any particular teaching approach, or innovation. This in turn raises the issue of assessment alternatives in mathematics education.

Towards More Authentic Criteria for Measuring Achievement in Mathematics Education Research

New Principles for Assessment in Mathematics Education

Although Clarke and Jasper's (1995) paper had the rather mundane title "A Practical Scheme for Classroom Assessment," much of what was in the paper looked forward to a redefinition of what constitutes effective mathematics teaching. Clarke and Jasper (1995) drew attention to four key elements of an emerging mathematics education agenda:

> The replacement of *measurement by portrayal* as the underlying metaphor for the purpose of assessment.
>
> The reconciliation of assessment and instruction, through the deliberate use of tasks which legitimately serve both purposes, and through the requirement that assessment both mirror and constructively inform instructional practice.
>
> The development of a new language of assessment by which the goals, practices, products, and consequences of the curriculum might be rendered coherently in both instructional and assessment contexts.
>
> The use of assessment as a catalyst for systemic reform in mathematics education. (p. 5)

In working towards achieving this agenda, Clarke and Jasper (1995) reported on how one of the authors (Jasper), a classroom teacher, used a range of assessment strategies which represented a clear change in practice from what was regarded as conventional classroom assessment. These "new" strategies were: (a) the annotated

classlist; (b) practical tests in mathematics; (c) student-constructed test items; (d) student self assessment; (e) student mathematics journals; (f) student mathematics portfolios; (g) student investigative projects; and (h) solutions to challenging problems in mathematics.

In a similar vein, Bishop (1993) argued that mathematics assessment programs should be as broadly permissive as possible and that they should therefore include, in addition to standard written tests, project work, investigations, essays, use of computer programs, models and other materials, and oral and practical work. According to Bishop examples of all of these types of assessment can be found in various parts of the world: what is lacking, though, is acceptance by "meritocratic" traditionalists (that is to say, most employers, parents, and government officials) that these should be used consistently throughout schooling, and indeed for both formative and summative assessment. Bishop (1993) added that he saw "no necessary reason why examinations in mathematics at school should be the same from one society to another" (p. 13), because each society should be free to make its choice in line with its political, educational and social agendas.

Triangulation

The range of methods of assessing mathematical learning evident in the lists provided by Clarke and Jasper (1995) and Bishop (1993) would seem to have immense implications for the teaching and learning of mathematics. There is a corollary, too, which is of particular relevance to this book: the instruments that mathematics education researchers choose to employ in their research should also incorporate a wide range of both receptive and expressive modes. In the new scheme of things, the traditional pencil-and-paper test should have much less status. In fact it should be just one of many approaches which would be used on a regular basis for assessing students' mathematical strengths, weaknesses, attitudes and preferences (Cobb, 1990b; Mousley, Clements, & Ellerton, 1992).

The idea of assessing learning and attitudes from a range of perspectives is called "triangulation" by qualitative education researchers. Stake (1988), for example in discussing a case study of learning in a computerised instruction environment using PLATO, told of how one researcher used data from interviews, notes taken in log books, direct observation, and other sources, to paint a holistic picture which left the reader in little doubt that the on-task behaviour of the pupils had been high throughout the year in which the study took place.

In the United States, the Mathematical Sciences Education Board (1993) has pointed out that current school assessment procedures for mathematics do not support, and in fact often work against, the kind of vision expressed by Clarke and Jasper (1995). According to the Mathematical Sciences Education Board (1993):

> For decades, educational assessment in the United States has been driven largely by practical and technical concerns rather than by educational priorities. Testing as we know it today arose because very efficient methods were found for assessing large numbers of people at low cost. A premium was placed on assessments that were easily administered and that made frugal use of resources. The constraints of efficiency meant that mathematics assessment tasks could not tap a student's ability to estimate the answer to an arithmetic calculation, construct a geometric figure, use a calculator or ruler, or produce a complex deductive argument.

> A narrow focus on technical criteria—primarily reliability—also worked against good assessment. For too long, reliability meant that examinations composed of a small number of complex problems, and examinees were asked to perform large numbers of smaller tasks, each eliciting information on one facet of their understanding, rather than to engage in complex problem solving or modelling, the mathematics that is most important. (p. 7)

In the absence of expressly articulated educational principles for the purpose of guiding assessment policies and practices, technical and pragmatic criteria became de facto ruling principles.

As Bob Stake (1995), the veteran American authority on education evaluation, has observed, teachers respond to increasing emphasis on accountability through testing and curriculum standardisation by attending more to the so-called basics (the most elementary knowledge and skills) and less to helping their students to achieve deep understanding. According to Stake, the dangers in current school reform, and especially the greater emphasis on pencil-and-paper statewide tests, are several: "overstandardisation, oversimplification, over-reliance on statistics, student boredom, increased numbers of dropouts, a sacrifice of personal understanding and, probably, a diminution of diversity in intellectual development" (p. 213). Stake (1995) was quite clear on the matter:

> Test scores are a stable ground for comparing schools and nations; unfortunately, those comparisons are often distractive, often pernicious. ... They can turn teachers and administrators to a lesser task. Comparisons make some deficiencies public but only on the rarest occasions do they show weaknesses not already recognized by teachers and representatives of the public. Seldom do they provide insight or diagnostic remedies for the deficiency. Nor are achievement test scores indicators of the quality of teaching. (p. 214)

The title of Stake's (1995) paper was "The Invalidity of Standardized Testing for Measuring Mathematics Achievement," and he could not have made his points more clearly. But his cry for more authentic assessment fell on deaf ears—the international community was too caught up in the process of establishing benchmarks in order that value-added system and school assessments might be achieved. Achieving so-called "quality" had become the order of the day, and the pencil-and-paper tests were still needed by education bureaucrats and administrators.

Kilpatrick (1993a) has commented that E. L. Thorndike's optimistic and strident claim, made over 50 years ago, that everything that exists can be measured—in some fashion—now almost seems to have been justified. Our comment would be that, of course, anything can be measured *in some form*. However, in education it is important to ask whether, in some situations one kind of form is better than another, and whether some forms are totally inappropriate. Regular use of a certain kind of measuring instrument might have the effect of facilitating desirable learning, whereas the reverse could be true of other forms.

We tend to agree with commentators who fear that pencil-and-paper tests have driven other forms of assessment out of schools (Kilpatrick, 1993a; Resnick, 1987), that they have not produced accurate data on what children really know, and that they have not assisted in the quest to produce more equitable educational

outcomes. Furthermore, in the realm of mathematics education research, we would argue that the overuse of standardised tests of mathematics has rendered much research less than useful, and has helped to maintain myths about the way "good" research should be conducted. This comment is particularly pertinent in many developing, or recently developed, nations in the Asia-Pacific region, where the idea still prevails that the only rigorous kind of education research is that which employs traditional experimental designs and standardised tests.

Although we have not dwelt on the issue of how language factors can have such a vital influence on the interpretations of, and therefore responses given to, pencil-and-paper short answer tests, a brief comment will be in order. The importance of the language factor was a feature of results obtained in a comparative study by Cai (1995) into the mathematics performance of 250 sixth-grade children in the United States of America and 425 sixth-grade children in China. Cai used English and Chinese forms of multiple-choice tests assessing, among other things, computation and simple arithmetic word problem performance. Cai found that although the Chinese students performed significantly better than U.S. students on both computation and simple arithmetic word problems, when subsets of U.S. and Chinese students were matched on their computational performance, the U.S. students scored significantly higher than comparable Chinese students on measures of both simple and complex problem solving.

Some of the largest comparative differences occurred on three "simple" arithmetic word problems which were primarily concerned with the use of the comparative polarised term "more" (Cai, 1995, pp. 142-146). For example, only 47% of the U.S. sample but 82% of the Chinese sample gave correct responses to:

9. Which number sentence is correct?

John has five more marbles than Pete.
(a) John's marbles = 5 + Pete's marbles
(b) John's marbles + 5 = Pete's marbles
(c) John's marbles + Pete's marbles = 5
(d) John's marbles = 5

Although Cai (1995) did not discuss the language factor in detail, the differences in semantic structures of the English and Chinese versions of this and other similar questions clearly contributed to the large differences in performance.

We shall allow Kilpatrick (1993a) the last word on assessment issues:

> The 20th century has produced an assessment practice in education that is dominated the world over by *psychometrics*, the measurement of psyche. The challenge of the 21st century, as far as mathematics educators are concerned, is to produce an assessment practice that does more than measure a person's mind and then assign that mind a treatment. We need to understand how people, not apart from but embedded in their cultures, come to use mathematics in different social settings and how we can create a mathematics instruction that helps them use it better, more rewardingly, and more responsibly. To do that will require us to transcend the crippling visions of mind as a hierarchy, school as a machine, and assessment as engineering. (p. 44)

7 Reconstructing the International Mathematics Education Research Agenda

Overview

This final chapter concludes our task of deconstructing and of tentatively reconstructing the international mathematics education research agenda. An attempt is made in the chapter to provide a critique of contemporary mathematics education research, especially from an Asia-Pacific perspective. The chapter concludes with a statement of ten propositions concerning present practice and future directions in mathematics education research. These ten propositions emphasise the need: (a) to give due accord to how linguistic and cultural factors influence mathematics education; (b) to question whether it is helpful to continue to accept as a basis for future investigations many of the existing "grand theories" which have been developed; and (c) to recognise the value of the wisdom of practice deriving from the classroom knowledge and action-oriented reflections and theories of practising teachers of mathematics.[57]

Eurocentric Perspectives in Mathematics Education

Most published accounts of mathematics education research investigations have been written by scholars from developed nations such as the United States of America, the United Kingdom, Japan, France, Germany, the Netherlands, and Australia. These nations have well-established education systems and are well placed to maximise the potential of hi-tech developments in education.

It is possible, however, that many developing nations have unwittingly accepted the research priorities inherent in the current mathematics education research agenda of developed nations, when arguably other more local issues need urgent investigation. Issues such as "What does a constructivist mathematics classroom look like?" or "How can we best use high technology to facilitate

57. Another version of Chapter 7 has been published in A. Sierpinska & J. Kilpatrick (Eds.) (1996). *What is mathematics education research, and what are its results?* Dordrecht: Kluwer Academic Publishers.

mathematical understanding in students?" may be important for mathematics education researchers attached to universities in Los Angeles (or Paris, or Sydney, or ...), but it is likely that the mathematics education research priorities should be different in developing nations such as Papua New Guinea.

The level of acceptance of the prevailing assumption that mathematics education research agendas which have been developed largely from Eurocentric models of mathematics, education, and mathematics education research, should be common across the world, would appear to be an example of educational colonialism at work. For, even within the same country, different research questions will need to be prioritised for different situations. That is why the principles of action research, as they have been outlined in Chapter 5 of this book, should be noted if the aim is to improve the quality of mathematics teaching and learning, both in particular schools and situations and in the overall educational system.

In fact, there is an urgent need to counter the persuasive rhetoric used by those with vested interests who support the idea of developing international curriculum frameworks, and common international research agendas. On occasions when we have made this point openly at meetings of senior educators in different parts of the world, the standard retort has been something like: "Surely, Pythagoras' Theorem is as true in the People's Republic of Vietnam as it is in France; Newton's Third Law holds everywhere, doesn't it? Why should developing nations waste precious resources in developing local curriculum materials, when more or less the same thing has been done by overseas experts, already?"

This kind of reasoning is flawed on two counts. First, we have carried out research which has shown that many of the glossy mathematics education materials produced in developed countries have not even been well regarded by teachers in the same countries where they were developed; exported versions to other nations (often supported by international aid programs) have been an abject failure (Ellerton & Clements, 1989b; 1994). Typically, the mathematical symbolism and notations used in the "expertly-developed materials" are not only different from those used in most schools in the nations receiving the materials, but also the mode of presentation is often culturally inappropriate.

And, second, if developing nations are seduced into importing "quality" mathematics curriculum materials and structures then this means that local educators have been denied the experience of developing the materials themselves. An attitude of dependency is thereby initiated, developed and nurtured.

In this respect, the following remark by Volmink's (1994) should be noted:

> My own cynicism is rooted in the observation that school mathematics curricula, in its canonical form, although extremely unlikely to be equally appropriate to people in all contexts are so remarkably similar all over the world. I believe that there are strong hegemonic forces at work in the global society that produces this resistance to change. (p. 62)

The ten propositions with which this chapter closes point in new directions for mathematics education researchers who recognise the need to work within the contexts of *local*, as distinct from Eurocentric, education perspectives.

Before stating these propositions, however, it will be useful to consider how and why each nation in the Pacific-Pacific region needs to develop its own

priorities for mathematics education research. This will be done from two perspectives: (a) education issues associated with the rapid changes in technology; and (b) certain features of the mathematics education scene in Indonesia.

Technology and the Brave New World in Education

In mathematics education the information technology revolution of the second half of the twentieth century has caused mathematics educators to contemplate seriously curriculum, pedagogical, and assessment issues associated with the increasing accessibility of calculators (including graphics calculators), CD-ROM, the Internet, and mathematics-related software such as LOGO, *Cabri Geometry* and *Mathematica*. At the level of professional development of teachers, interactive systems, including video conferencing, are being employed to assist teachers to cope with high-tech interactive systems. Many teachers, however, feel threatened by the pace of change.

Flexible Delivery Systems in Education

In 1994 in the State of Victoria (Australia), for example, all government primary and secondary schools were fitted with satellite dishes and decoders to access curriculum by interactive satellite television—a new form of education at a distance. The network, called "SOF Net," is used to disseminate programs in three categories: curriculum, corporate information, and teacher professional development. It is intended that in Victoria, systems of computers will be installed in all classrooms of government schools (Peck, 1995).

An article published in a Malaysian daily newspaper, the *New Straits Times*, on August 23, 1995, informed readers that sixth-grade school children living in remote parts of Iowa, in the United States of America, were participating in education programs which enabled them, through the use of advanced fibre-optic networks, to provide interactive voice and video links between classrooms and instructors hundreds of kilometres apart. The children were talking directly with NASA rocket scientists and engineers.

Technology and Distance Education in the Asia-Pacific Region

How close are we, in the Asian-Pacific region, to delivering hi-tech, multimedia, quality distance education programs? The answer is clear: we are not just close—in fact, we are actually there. For several years before we left Deakin University (in 1993) communication via video conferences between staff at the five campuses of that multi-campus University was commonplace. In April 1995, those attending a conference held at Monash University on "Regional Collaboration in Mathematics Education" joined with educators physically located in the United States (attending the America Education Research Association Conference), in a videoconference on the theme "Assessment Alternatives in Mathematics Education."

While working on the Internet recently, we could not help but notice the

number of advertisements for "paper-free" academic courses. Prospective students are being told that all they have to do is to enrol in a course, and they will then be able to qualify for higher degrees without ever having to put pen or pencil to paper. The enrolment itself can be completed electronically, course fees can be paid electronically, the syllabuses and text "books" can be communicated electronically, and all assessment tasks can be submitted electronically. Comments by lecturers who assess students' submissions can be conveyed to the students by electronic means. Furthermore, throughout such courses students are able to interact with each other and with their lecturers electronically.

Educators in all nations within the Asia-Pacific region are faced with the question of how best to capitalise on the range of technologies now available to them. The titles of many books and papers published in the region reflect their desire to make creative use of the possibilities, particularly through flexible delivery modes, to improve the quality of teaching and learning (see, for example, Hanizah & Sahol, 1996; Ismail, Selamat & Pahri, 1996; Nebres, 1996; Rudra, 1996; Serudin & Sim, 1996; Sutabutr, 1996; Wang, 1996)

Colonialism through distance education. It makes no sense to provide education institutions in developing countries with hi-tech, expensive equipment when this is inappropriate (Napitupulu, 1995). Equipment which cannot be properly used or maintained is an embarrassment to all concerned. It may well be wiser to attempt to meet local needs by introducing well designed, traditional distance courses which are mainly based on the print medium. It is the experience of institutions such as the Open University, in the United Kingdom, and a number of Australian universities, that effective distance education programs can be developed which do not depend on access to advanced technology.

In 1995 the Australian higher education weekly, *Campus Review*,[58] reported the results of a survey carried out by the Queensland University of Technology which indicated that, despite the availability of new technologies to aid teaching strategies, most lecturers in Australian tertiary institutions continue to use traditional "chalk and talk" methods. According to report, the most used piece of technology was clearly the overhead projector. Little use was made of laser disc players, electronic visualisers, slide projectors, videotape recorders, and modern software such as Powerpoint.

Given that this is the case in a developed nation, where equipment maintenance and repair by highly trained technicians are readily available, personal computers are commonplace, and educators have ready access to inexpensive or free training programs in the use of technology, one is forced to wonder about the utility and logic of hi-tech equipment being liberally provided to developing nations through "generous" aid packages.

Lessons From Indonesia

Some important lessons for mathematics educators have emerged from recent developments in Indonesia, a nation of 7000 islands and 200 million inhabitants.

58. *Campus Review*, September 14–20, 1995, p. 26.

Although there are about 300 indigenous languages and indigenous number systems[59] in Indonesia, the national government has chosen to have *Bahasa Indonesia* as the one national language. *Bahasa Indonesia* is the language of instruction in schools.

Historically, when Indonesia was proclaimed an independent nation in 1945, only a proportion of the population had ever attended school, even at the junior primary level. From the beginning of the 1970s, concerted efforts were taken to improve the education profile of the nation. As a result, access to primary schooling increased from 50% in 1968 to 93% in the mid-1990s (with a total enrolment of 29 million pupils).

Secondary school enrolment in Indonesia increased from about a 10% participation rate in 1975 to about 40% in 1995. As a result of the rapidly expanding student enrolment at the secondary level, in the early 1980s more than 700 000 secondary school graduates sought admission into tertiary education institutions in the nation, which could accommodate no more than 250 000 students.

The Indonesian Government responded to the demand in 1984 by establishing the Indonesian Open Learning University. In 1995 this University had an enrolment of almost 400 000 students, living in scattered places across the archipelago. More than half of the students are primary school teachers, seeking to upgrade their qualifications and knowledge. In fact, in 1993 there were about 1 200 000 primary school teachers in Indonesia, the majority of whom have had only a secondary school education. Many of the schools are in remote areas of the nation, and the Government is determined to provide professional development for the teachers. One of the major programs is a radio broadcast diploma program known as "D2 Air." This has four components: printed modules, science kits, video cassettes, and the radio programs. Teachers enrolled in the program are expected to study the modules, and listen to the radio programs each day, Monday through Saturday (Purwanto, 1995).

The rapid expansion of primary education in Indonesia has placed massive pressure on secondary schools, and in 1979 five "open junior secondary schools" (called *SMP Terbuka*) were established to provide the first three years of secondary education to those children for whom there was no place in the existing secondary schools or who, for whatever reason (often the need to supplement family incomes) could not attend during the regular school day. By 1994 there were 59 *SMP Terbuka*, and it is planned that by the year 2010 there will be 13 000 *SMP Terbuka* catering for well over two million students (Syarief & Seligman, 1995).

SMP Terbuka Schools follow the same curriculum as the regular junior secondary schools of Indonesia, but the students learn from printed modules, have special study centres which they attend on four or five days a week where their study is supervised; the students attend a regular school once a week for tutorials. Although the system is centrally organised, there is considerable community involvement through the provision of local resources, study centre supervisors, and specialist subject teachers. Community acceptance is high, and a growing

59. The discussion of Indonesian education is based on details given in Suprato Brotosiswojo (1995).

number of *SMP Terbuka* graduates proceed to senior secondary school (Sadiman, Seligman & Rahardjo, 1995; Syarief & Seligman, 1995).

It is foolish, however, to assume that solutions to education problems in one country or context are appropriate to other countries or contexts. Syarief and Seligman (1995), in commenting on efforts to provide hi-tech answers to the problems of increasing participation in junior secondary education in Indonesia stated:

> Looked at from [the perspective of] a developing country it appears to be a good solution—for somebody else. It is too hi-tech, too expensive and totally inaccessible to the vast majority of children, even if the infrastructure for provision was there.

The same point was made, graphically, in another paper by Hareng, Ali, Sadjad and Johari (1995), who described education systems in Eastern Indonesian States.

In the mid-1980s, a USAID-assisted project called the "Rural Satellite Project" determined to use an interactive audio-conferencing system together with an interactive graphic system and a dial-up telephone system to assist Eastern Indonesian universities to reach students in more effective ways. However, according to Hareng et al. (1995):

> The system was totally designed without any involvement of the local technical team. The original designed system consisted of pieces of equipment granted by USAID which turned out to be inappropriate choices for the environment. Thus, in the later phase, we are forced to redesign the system to use full indigenous technology, appropriately. From experience we learned that local equipment ... has been more appropriate (p. 4).

In fact, the original system lasted for a year only—the graphic system did not function at all—before a local technical team developed new equipment to meet the specific needs of the project.

Voice Scholarship

Kitchen (1995) urged mathematics education researchers around the world to adopt what he called "voice scholarship" approaches. The intent of voice scholarship is specifically to provide an avenue to hear from those people who traditionally have not been heard. Such scholarship can contribute to an improved understanding of what knowledge has been imported from the First World, while simultaneously suggesting possible alternative models of mathematics pedagogy that may be appropriate in different cultural contexts. Kitchen (1995) argued, powerfully:

> Voice scholarship also affords a way to hear from teachers working in places that have so little of even the basic amenities that First World teachers take for granted (e.g., blackboards, chalk, books, and paper). My review of the literature has revealed very little research that allow actual mathematics teachers working in peripheral nations to describe the context of their working conditions in their own words. (p. 2)

At the same time as drawing attention to the potential strengths of the "voice scholarship" methodological approach, we feel obliged to point to inherent

weaknesses. In particular, it needs to be recognised that the researcher who designs and implements research which allows the voices of teachers to be heard is usually the same person who collates, analyses, interprets and reports the data. Regardless of the intentions and integrity of the researcher, such a filtering process inevitably leaves its mark not only on the data, but also on what is reported as the "results" or "findings" of the research.

The Eastern Indonesian case, cited above, of foreign aid allowing hi-tech "solutions" to be introduced in contexts where this was totally inappropriate, draws attention to the need for mathematics education researchers to develop new research methodologies which enable the voices of stakeholders to be heard and evaluated, without the power of the dollar—or the pound—becoming too influential.

We would maintain that, regardless of the intentions of donors, the granting of funds for hi-tech education projects which are not appropriate to the needs of local educators constitutes a colonialist act. Local educators should be involved in all decisions which are likely to affect the provision and quality of education services in their region (Berman, 1992; Secada, 1995). That, we believe, is the message of history. It is also totally consistent with the principles of action research and with the findings of researchers who have evaluated the effectiveness of various professional development programs for teachers (Robinson, 1989).

The point being made has been beautifully expressed by Fasheh (1989), a Palestinian mathematics education researcher, who, in discussing the difference between the Western mathematics that he studied and taught and the mathematics that his mother used on a daily basis, commented:

> My math had no power connection with anything in the community and no power connection with the Western hegemonic culture which had engendered it. It was connected solely to symbolic power, and without the official ideological support system, no one would have "needed" my math; its value was derived from a system of symbols created by hegemony and the world of education ...
>
> Math was necessary for her [his mother] in a much more profound and real sense than it was for me. My illiterate mother routinely took rectangles of fabric and, with few measurements and no patterns, cut them and turned them into beautiful, perfectly fitting clothing for people. In 1976 it struck me that the math she was using was beyond my comprehension. Moreover, while math for me was a subject matter I studied and taught, for her it was basic to the operation of her understanding. In addition, mistakes in her work entailed practical consequences, completely different from mistakes in my math. (pp. 84–85)

Yet, Fasheh's mother had obviously been happy for her son to be educated in the ways of the West.

Henderson (1996), who for twenty years has taught geometry courses to students taking mathematics majors at Cornell University, made a similar point in a different way. He argued that most "well qualified" mathematics lecturers are so full of traditional ways of thinking about mathematics, and of standard ways of proving mathematical relationships and theorems, that they simply do not "hear" the interesting and often very creative approaches coming from female students or from students from non-European backgrounds. Henderson (1996), presented data

to support three conclusions which he had reached about how he, himself, tended to react to the mathematical efforts of his students:

> *Conclusion 1:* In order for me to be satisfied by a proof the proof must answer my why-question and relate my meanings of the concepts involved. (p. 48)
>
> *Conclusion 2.:*A proof that satisfies someone else may not satisfy me because their meanings and why-questions are different from mine. (p. 48)
>
> *Conclusion 3:* Persons who differ the most from me (for example in terms of cultural background and gender) are most likely to have different meanings and thus have different why-questions and different proofs. (p. 49)

Henderson (1996) added a corollary that he could learn much about mathematics if he listened to persons whose cultural backgrounds or gender were different from his. He recognised that considerable effort and patience on his part was required for him to "hear" someone else's proofs. Yet, in an intriguing table, he presented data which suggested that it was white female students and non-white male students who had shown him some mathematics which was new to him.

Most tertiary-level mathematicians are less patient, and reflective, than Henderson, however, and the result is that students, in order to survive in a so-called academic mathematical environment as long as possible, learn to construct "standard" mathematics (Gerofsky, 1996). Ellerton (1989) asked a large number of secondary students to write a difficult word problem, and she found that overwhelmingly they attempted to write problems which were similar in form and content to those in their textbooks. Lave (1992) claimed that if children were asked to make up problems about everyday mathematics they do not make up problems about their experienced lives, but rather they present problems in the genre of the textbook—As Lave (1992) commented, "they too know what a word problem is" (p. 77).

Towards a Wider Perspective for Mathematics Education

The reason why, earlier in this final chapter, we provided details concerning some aspects of education developments in Indonesia is that we wanted to establish our case that most mathematics educators and mathematics education researchers living in Western nations and in other developed nations would find it difficult to imagine the circumstances in which mathematics is being taught, and learned, in many parts of the world. Similar, yet in fact very different, descriptions of education situations in India, China, Bangladesh, Pakistan and other highly populated nations could have been used to illustrate our perspective

It would be foolish, even arrogant, for members of the international mathematics education research community, researching from the largely Eurocentric mathematics education perspectives which characterise research papers published in international mathematics education research journals, to believe that their work provides a sound basis for studying mathematics education. ... all countries. It is likely that the factors which influence how mathematics is

taught and learned in schools in the cities and villages of nations such as China, India and Vietnam are quite different from those which influence mathematics education in Western nations. Yet, of course, it is well known that the nations of China, India and Vietnam have produced many excellent mathematicians. This fact makes it all the more difficult to know how and where to start in trying to establish agreement—if indeed there should be any agreement—on how mathematics education issues which cross national boundaries should be researched.

Both of the authors of this book are mathematics education researchers who regard themselves as members of the international mathematics education research community. Having said this, we would wish to assert that we are not convinced that many of the current emphases in mathematics education research are more than tangentially relevant to the teacher of mathematics in a remote school in an isolated region of Papua New Guinea, Indonesia, or China.

We are convinced that mathematics education research needs to take more account of the influence of difficult linguistic, cultural, and equity variables. We would wish to say, bluntly, that current moves towards an internationalised mathematics curriculum (Oldham, 1989) make little sense if reviewed through the twin lenses of cultural and linguistic differences. We believe that participatory action research is urgently needed to prevent technological "advances" from leading, seemingly inevitably, to forms of mathematics education which will exclude many students from quality education experiences. Without this approach, students' rights will be compromised, and this will only increase the gap between the "haves" and "have nots."

Furthermore, we are not convinced that a single epistemological framework (for example, that associated with radical constructivism) can be appropriate in all education contexts. It is more likely that a whole range of epistemologies will emerge, with different epistemologies evolving for different cultures.

Questioning the Agenda and Methods of Contemporary Mathematics Education Research

Mathematics Education Research Under the Microscope

During the second half of the 1980s the international mathematics education research community was prepared to ask fundamental questions about the major issues and methods of its field of endeavour (see, for example, Charles & Silver, 1989; Grouws, Cooney & Jones, 1989; Hiebert & Behr, 1988; Sowder, 1989; Wagner & Kieran, 1989). This questioning has continued into the 1990s, with mathematics education researchers around the world scrutinising the assumptions and methodologies associated with their work (see, for example, Kilpatrick, 1992, 1993b; Sierpinska, 1993; Mason, 1994; Secada, Fennema, & Adajian, 1995; Sierpinska & Kilpatrick, 1996).

One manifestation of this interest in the foundations of mathematics education research was a symposium on "Criteria for scientific quality and relevance in the didactics of mathematics" held at Gilleleje, Denmark, in 1992. Another was an International Workshop on the theme "What is mathematics education research and what are its results?" which was sponsored by the International Commission

on Mathematics Instruction (ICMI) and held at the University of Maryland in May 1994. Further evidence for the upsurge in interest in mathematics education research comes in the form of a well-attended Working Group on "Criteria for quality and relevance in mathematics education research" which met at the 8th International Congress on Mathematical Education (ICME 8) in Seville, Spain in July 1966.[60]

At the Gilleleje Symposium, papers by Kilpatrick (1993b) and Sierpinska (1993) identified eight criteria which have been used to evaluate the scientific quality of education research in general, and mathematics education research in particular: relevance, validity, objectivity, originality, rigour and precision, predictability, reproducibility, and relatedness. Both papers were concerned with discussing whether these should continue to be fundamental criteria for assessing mathematics education research.

Despite the willingness of mathematics education researchers to put their work under the microscope, we believe that the debate over the agenda and methods of mathematics education researchers has been too conservative. In the mid-1990s around the world well over one billion humans are attending formal mathematics classes—in schools and other educational institutions—in which they are being taught the internationalised version of mathematics. A massive amount of money (probably of the order of US$1000 billion, annually) is spent on formal mathematics education. Yet, the international mathematics education research community seems to have only rarely paid attention to the fact that many children around the world are still required to sit in classrooms where they understand only partially the words and symbols being used by teachers and by textbook writers.

This book calls for a radical reconstruction not only of the international mathematics education research agenda, but also of prevailing attitudes towards what constitutes good research. It acknowledges the tenor of Skovsmose's (1994a,b) view that mathematics education, as both a practice and a form of critical research, should not only seek to do away with oppressive practices, but should also become a vanguard for reform by putting forward alternative, potentially liberating concepts and viewpoints. According to Skovsmose (1994a), mathematics education researchers should discuss "basic conditions for obtaining knowledge, ... be aware of social problems, inequalities, suppression, and ... [be] an active progressive social force" (pp. 37–38).

Yet, much of the contemporary theorising and practice by mathematics education researchers continues to take place as if mathematics education itself is, and should be, a neutral, value-free activity. We contend that mathematics education researchers should play a vital role in the creation of more equitable forms of mathematics education around the world. In short, mathematics education researchers need to recognise that their research ought to be both applicable and transformative.

60. Both authors were present and participated in the Working Group, which met for three ninety-minute sessions. In fact, the first two of the three sessions were dedicated to critique and discussion of issues raised in a paper written by the authors (Ellerton & Clements, 1996a). Most of the issues raised in that paper are also raised in this present chapter.

Key Questions Faced By the International Mathematics Education Research Community

A background paper (Balacheff, Howson, Sfard, Steinbring, Kilpatrick & Sierpinska, 1993) was prepared in order to stimulate and direct the thinking of those who contributed to the 1994 Maryland Workshop. Five key questions were raised:

1. What is the specific object of study in mathematics education?
2. What are the aims of research in mathematics education?
3. What are the specific research questions or *problématiques* of research in mathematics education?
4. What are the results of research in mathematics education?
5. What criteria should be used to evaluate the results of research in mathematics education?

These questions beg other related questions. For example, *should* there be a specific object of study in mathematics education? Would it not be reasonable to expect that the aims, and indeed the *problématiques* of mathematics education research, should depend on historical, cultural, economic, and local circumstances? And, since criteria for evaluating results of research in mathematics education have not been agreed upon, is it possible to be objective about precisely what the results are?

Bishop's (1992) approach was slightly different in that he identified the following five critical relationships in research in mathematics education:

1. "What is" and "what might be";
2. Mathematics and education;
3. The problem and the research method;
4. The teacher and the researcher;
5. The researcher and the educational system.

These critical relationships would seem to call for more attention to be given by mathematics education researchers to localised influences on mathematics education.

What might happen if more attention *were* given to localised influences on mathematics education? Perhaps we should allow Joseph (1987) to speculate on that question. He called for the development of new ways of thinking which would serve as a springboard for mathematics curricula, classroom structures, teaching methods, and research investigations which would challenge the following four key, longstanding assumptions which have affected the progress and forms of both mathematics and mathematics education:

1. Mathematics is a form of absolute truth, and its properties and relationships are pre-existent, merely "waiting to be discovered." Therefore, in its purest form, mathematics does not have a materialistic base, and in that sense is free of economic, political and cultural forces.

2. The discovery and investigation of mathematics should be left to those relatively few who possess the requisite intellectual qualities and gifts.

3. Mathematical truths can only be confirmed by the rigorous application of deductive axiomatic logic believed to be a unique product of Greek mathematics.

Intuitive or empirical methods are not sufficient for proof, and hence have only peripheral relevance in mathematics.

4. Mathematicians tend to believe that the presentation of mathematical results must conform to the formal and didactic style devised by the Greeks over 2000 years ago. Their practices suggest that they also believe it is proper to confine the processes of validating and disseminating new additions to mathematical knowledge to a small, self-selecting, largely Eurocentric coterie.

Benn and Burton (1996) have argued that Eurocentric biases have not only shaped the way mathematics has been constructed, but have also infused twentieth century mathematics education with unfounded élitist, racist and sexist, modes of operations. If, somehow, mathematics educators were able to free themselves from the shackles of a world view which restricts the higher pursuits of mathematics to a privileged minority, then a new vision of mathematics education might emerge. Mathematics, seen from the widest possible ethnomathematical perspective, should be for *all* people, everywhere.

One wonders what mathematics education research might look like then. Burton (1994) has stated that it will only be by moving mathematics pedagogy into the same world of conjecture, relativism, and incompleteness which matches the fallibilist view of mathematics that we will have a chance of breaking the image of mathematics as European and male and of creating the kind of classroom in which all students can find excitement, interest, and success. According to Burton (1994):

> It seems to me to be a supreme irony that students disengage from the subject known as school mathematics which itself is rejected as an adequate representation by many professional mathematicians. Were mathematics to be experienced as a searching, hesitant, intuitive area of study, open to interpretation and challenge, I feel confident that there would be a much greater identification with its style and ideas, by pupils from both sexes, different classes and different races. (p. 79)

Burton (1994) calls for a radical reconceptualisation of school mathematics, with mathematics education researchers leading the way in the process of providing suitable bases for sensible curriculum and pedagogical changes.

An Example of a Recent Mathematics Education Research Study Which Does Not Fit the Mould

When we read Kilpatrick's (1993b) and Sierpinska's (1993) papers—delivered at the Gilleleje Symposium in Denmark—we felt uncomfortable about their eight criteria (relevance, validity, objectivity, originality, rigour and precision, predictability, reproducibility, and relatedness) for evaluating mathematics education research. We felt uncomfortable because we had only recently completed a major investigation which, although definitely within the category of "mathematics education research," did not rest easily with the eight criteria.

Our research has been reported in detail in a 400-page book entitled *The National Curriculum Debacle* (Ellerton & Clements, 1994). The book provides a history of an attempt, over the period 1987–1993, by Australian politicians and education bureaucrats, to establish a national curriculum. We extensively documented our argument that consultative processes adopted by the politicians and bureaucrats were inadequate. The outcomes-based curriculum framework

which was developed and expressed in the nationally developed school mathematics documents was not acceptable to leading mathematicians, mathematics educators and mathematics teachers in Australia. In the book, we argued that outcomes-based forms of education have their origins in behaviourism, and claimed, on the basis of existing literature, that behaviourist approaches to mathematics education have rarely been associated with quality mathematics teaching and learning.

One of our concerns about the eight criteria listed by Kilpatrick and Sierpinska is that any attempt to apply them to historical research of the kind that we conducted would appear to be inappropriate. Certainly, we would hope that our research was relevant and valid, but we would find it difficult to define what the words "relevant" and "valid" mean in the context of our research. So far as "objectivity" is concerned, in the "Foreword" to our book, we acknowledge that there is no such thing as "objective" history. Despite Kilpatrick's (1993b) point that researchers should strive for objectivity even if they know they cannot achieve it, we wish to say that in the domain of investigation covered by our research it is difficult to know what "objectivity" might look like. In addition, it would be difficult to apply the expression "rigour and precision" to our research (although we believe that our investigations were comprehensive and thorough).

Regarding reproducibility, we recognise that although someone might seek to replicate our research, the perspectives which we brought to bear on the various issues raised in the book were idiosyncratic. No-one else would have chosen to carry out, or could now carry out, the research in exactly the way we did. From that point of view the issue of "reproducibility" would seem to be irrelevant so far as our historical investigation was concerned. Certainly, the documents which we examined and discussed are still available, but for most historians the idea of someone else replicating research is not regarded as sensible. Documents can be checked, but the interpretations of documents and the marshalling of evidence which might persuade a reader to accept suggested causal links renders the idea of replicating a historical study as nonsensical. That is not what good historians do.

Our last sentence ("That is not what good historians do") suggests that mathematics education research can, for example, be historical research, and that if it is historical research then criteria for evaluating the quality of historical research must apply. Of course, all scholarly historians do not agree on the criteria which apply to their domain, a fact which further suggests that any attempt to specify criteria for judging the quality of mathematics education research must inevitably result in failure. Mathematics education research can be in the realm of history, psychology, anthropology, linguistics, sociology, philosophy, mathematics, etc, and there is not within-domain or between-domain agreement on criteria for evaluating research. Hence, we must conclude that any attempt to introduce more than loose criteria, or guidelines, for assessing the quality of mathematics education research, would be foolish.

Probably, *The National Curriculum Debacle* would fit both Kilpatrick's and Sierpinska's criterion for "relatedness." But, as has been argued above, most of the eight criteria do not really match the research that we carried out. The criteria echo lists of criteria found in treatises on how to conduct "scientific" research. Our research was of an historical nature.

Ten *Problématiques* for Mathematics Education Research

Although it would be an unprofitable exercise to attempt to define specific generic criteria for assessing the quality of mathematics education research, it will be helpful to identify a set of important *problématiques* of mathematics education. Any list of "common" concerns will inevitably reflect the backgrounds of those who formulate them: nevertheless, the existence of such a list might provoke mathematics education researchers to generate related research questions and to consider which methods should be used to investigate them.

We now put forward ten possibly contentious propositions which define *problématiques* of mathematics education research.

1. Identifying the Bases of Current Practices in School Mathematics

Despite claims by some historians that "the nineteenth century had solidly established universal primary education" (Connell, 1980, p. 4), in 1900 less than 10 per cent of the world's adult population had attended elementary schools for more than one or two years. Furthermore, only 1 per cent of adults had ever attended a secondary school. In most countries, school mathematics programs were based on rigid, externally prescribed curricula, formal textbooks, and written examinations, and it was accepted that only the best students should go far in the formal study of mathematics (Ellerton & Clements, 1988).

Because of the steady increase in world population, and the dedication and persuasiveness of some who believed that all people everywhere have a right to acquire basic literacy and numeracy skills, the twentieth century witnessed a rapid escalation in the demand for more schools and more trained teachers. As the century progressed, bodies such as UNESCO, the World Bank, and the Asian Development Bank, as well as international aid organisations attached to wealthy industrialised nations, exerted pressure on governments of developing nations to provide an elementary education for all children. In developed countries more and more people gained a secondary education. This phenomenon generated important curriculum and associated teacher-training and professional development questions (Adler, 1996; Bishop, 1993;Volmink, 1994).

The time has come to stop and think, to reflect on what has been achieved, and what has gone wrong in the course of such rapid development. Our first proposition calls for the identification of the underlying assumptions from which current mathematics curricula and "school mathematics cultures" have been developed. Implicit in this proposition is the question: What can be done to reduce, and ultimately eliminate, the prevalence and force of unwarranted assumptions?

> *Proposition 1:* Many outdated assumptions influence the way school mathematics is currently practised. The identification of those assumptions which most urgently need to be questioned represents the first, and perhaps most important, *problématique* of contemporary mathematics education research.

The world has moved from the industrial age to the information age, and it is appropriate, indeed important, to reconsider what should be the content and priorities for mathematics curricula, and how these should differ depending on culture and context (Cockcroft, 1994; Davis, 1994; Ellerton & Clements, 1988; Nebres, 1996; Noddings, 1994; Usiskin, 1994a).

Many of the assumptions which have, up to now, influenced normal patterns of behaviour in mathematics classrooms (represented by teaching methods, assessment methods, and the attitudes of students, teachers, school administrators, employers, parents, etc.) were inherited from nineteenth century patterns of schooling in leading Western nations such as the United States of America, England, France, and Germany. These assumptions, and corresponding expectations for what school mathematics programs should achieve, are likely to be hopelessly inadequate for the twenty-first century.

The effectiveness of traditional mathematics classroom practices are being challenged, and alternative approaches such as problem-based learning (see Hiebert et al., 1996), are being put forward. According to Hiebert et al. (1996), "treating mathematics as problematic requires changing the entire system of instruction" (p. 19). Certainly, mathematics education researchers need to take more account of the influence of affective factors on mathematics learning, and we need to gain a better understanding of how these factors are influenced by societal, cultural and classroom forces. The importance of "new" variables, such as student "responsiveness" (Owens, 1993), might also be profitably explored.

A strong case could be made for asking governments to attempt to introduce policies aimed at achieving large-scale, even radical—but nevertheless research-based—changes in mathematics education practices.

There are longstanding problems which need to be solved. Consider for example, the issue of gender imbalance in participation in senior secondary and tertiary mathematics programs. In the mid-1980s, for every seven males attending African universities there was just one female. Women were particularly poorly represented in mathematics courses, in the physical sciences, and in engineering courses (Graham-Brown, 1991). Similar patterns have been reported in Papua New Guinea (Kaely, 1995) and Singapore (Kaur, 1995b). Even in the mid-1990s the gender imbalance in participation in high-level mathematics courses still remains in Western nations (Leder, Forgasz, & Solar, 1996). The question, apparently defying solution, is how such a situation can be rectified.

It is important that the questioning process occurs within a climate which allows *anything* to be questioned. Nothing should be allowed to be sacrosanct. For example, it was long assumed that structured materials, such as Dienes' Multibase Arithmetic Blocks, more or less "embodied" certain important place-value numerical notations and concepts, and were excellent teaching and learning aids which should be found in all primary school classrooms. However, such a view has been subjected to scrutiny by researchers (see, for example, Pimm, 1995), who have argued that in assessing the educational value of these blocks one needs to recognise that there is nothing inherently mathematical in the blocks themselves, and that in order to realise how they are linked to place-value concepts one needs *already* to understand the concepts. Similar arguments can and have been presented h respect to all physical "manipulatives" (Boulton-Lewis, Wilss & Mutch, 1996; ɔb, Yackel, & Wood, 1992).

One must go further and ask whether it makes much sense to assert that there are *core* mathematical skills which are "basic" for *all* people, and certain generic teaching approaches which will maximise learning in *any* mathematics classroom at *any* place at *any* time. To illustrate the point being made, consider the case of Indonesia. In the second half of the 1990s less than half of the nation's school-aged children are receiving a secondary education, and only about one-tenth enrol in any form of higher education (Suprapto Brotosiswojo, 1995). Under such circumstances one wonders whether school mathematics curricula in Indonesia should be designed mainly to meet the needs of those who will go on to study mathematics and/or science in higher education institutions. What kind of mathematics curricula are needed for the nation's schools? Given the dramatic expansion of formal education provision in Indonesia, what kinds of initial teacher-education and professional development programs are needed for primary and secondary teachers to enable them to cope with the demands of the mathematics education programs being offered in the nation's schools?[61]

Not a few contemporary mathematics education researchers have set themselves the task of identifying and connecting powerful sociocultural forces which operate in school mathematics classrooms. For example, Walkerdine (1988) has explored how children "come to read the myriad of arbitrary signifiers—the words, the gestures, objects, etc., ... with which they are surrounded such that their arbitrariness is banished and they appear to have the meaning that is conventional" (p. 3). Brown (1996) used a phenomenological approach to describe how individuals confront and work with mathematical ideas. He has argued that it is an individual's experience of grappling with social notation within his/her social situation, which is the major factor influencing mathematics learning.

The above discussion suggests that a word of warning is needed concerning this first proposition: it is unlikely that the set of unwarranted assumptions would be culture-free. In other words, findings with respect to *Proposition 1* are likely to vary, not only from nation to nation but also from culture to culture.

2. Doing More Than Prepare Students for the Next Highest Level of Mathematics

Proponents of national curricula in the United States of America, the United Kingdom, and Australia have tended to argue that there should be a set of core outcome behaviours in key learning areas that all learners should be able to perform. They also claim that these outcomes can be placed in "levels." It has been confidently asserted that the best way for "mathematics for all" to be achieved is to expect all school children to learn the same core mathematics, with advanced enrichment activities being made available to those with a special aptitude and/or interest in the subject. Topics at one level are being linked with those at the next level to form a mathematics curriculum which is unashamedly based on the assumption of "hierarchy" (Australian Education Council, 1991; Curriculum Corporation, 1994; National Council of Teachers of Mathematics Commission on

61. Note that in 1995 more than 200 000 practising teachers were enrolled in teacher education programs being offered by the Indonesian Open Learning University (Suprapto Brotosiswojo, 1995).

Standards for School Mathematics, 1989; Noss, 1994).

The assumption that mathematics comprises a hierarchical set of knowledge, skills, relationships and principles which can be sensibly linked with a hierarchy of student outcome statements is currently one which mathematics curriculum developers and teachers in many countries are being asked to accept. It is the responsibility of researchers to question whether this assumption *is* reasonable. A related issue arises from the belief, held by most university executives and staff in mathematics departments, that it is reasonable to expect their first-year students to be able to do the same kind of mathematics, at more or less the same level, as students in the past.

Many universities have introduced "bridging programs" to bring "deficient" students to an "acceptable" level; others have chosen simply to fail greater proportions of students. The possibility that the old first-year tertiary mathematics programs developed in Western universities may no longer be appropriate in the same universities has hardly, if ever, been taken seriously. The further possibility that such programs may not meet the needs of universities in developing nations should be examined.

Attempts by school systems to reform mathematics curricula have often been opposed by tertiary mathematicians on the grounds that new courses would not prepare students as well as the old for first-year tertiary mathematics courses. That kind of backward-looking thinking is not what is required for the 21st century.

This is the basis for the second proposition.

> *Proposition 2:* Those concerned with mathematics education need to develop ways of investigating claims that: (a) mathematical knowledge, skills, relationships and principles are characteristically hierarchical; and (b) the main concern of school mathematics is preparation for higher level mathematics courses.

In particular, the assumption that the quality of mathematics programs in schools should be judged by how many students go on to study tertiary mathematics (or by how well those who proceed to tertiary mathematics perform in mathematics examinations at a university), needs to be scrutinised.

3. *Making Language Factors a Central Concern*

Our third proposition is in line with comments made by Secada (1988) on how cultural minorities and students in many so-called developing countries have often been tacitly regarded as deviant and marginal in mathematics education research. Secada pointed out that in many parts of the world it is normal to be bilingual, yet most mathematics education research has been with monolingual children—with bilingual children being regarded as likely to need special attention.

And although in some countries there have been a series of successful court challenges to the right of governments to make compulsory a particular "official" language as the language to be used for instruction in all government schools (see, for example, Riffel, Levin, and Young, 1996), language factors continue to influence greatly the teaching and learning of mathematics in all nations in the Asia-Pacific region (McLaughlin, 1995).

In fact, in many schools, including schools in Western countries, there are children (in some cases, a majority of children in a school) who are required to learn mathematics in classrooms where the language of instruction is not their first language (Riffel, Levin & Young, 1996). Sometimes young children sit in mathematics classrooms for years understanding very little of what the teacher or textbook writer or examiner is saying. As Khisty (1995) has stated:

> The teaching and learning process consists of an interaction between persons for the purpose of developing and sharing meanings. It logically follows that language is crucial if the development of meaning is to occur. Consequently, if we are to fully understand instructional dynamics—and obstacles that arise in the process and constrain minority children—we must examine not only curriculum and classroom activities, but also classroom discourse, that is, what is said and how it is said. (p. 280)

Ironically, sometimes it is claimed that in order to achieve "equality of educational opportunity" the language of instruction and the curriculum should be the same for all, or most, children attending schools in the same country.

This leads to our third proposition:

> *Proposition 3:* The implications for mathematics education of the fact that many mathematics learners are bilingual or even multilingual urgently need to be explored.

In addition to issues associated with bilingual and multilingual learners a whole range of linguistic factors impinge on mathematics learning (Cocking & Mestre, 1988; Ellerton & Clements, 1991a). Although much has been done in regard to identifying and relating these factors, more research is needed. In fact, we would wish to assert that the whole issue of how language factors affect mathematics learning has been, for too long, on the periphery of the international mathematics education research scene. It should be centre stage.

Consider, for example, the situation in the nation of Papua New Guinea, where more than 750 languages are spoken. Until 1995, English was the language of instruction for all grades in PNG schools, which meant that many young children sat in classrooms for several years not understanding the language of the teacher and the textbook (Clements & Jones, 1983). Now, however, the official policy is that, in Community Schools, the language of instruction is the mother tongue of the learners. Thus there are over 750 official languages of instruction. A similar dilemma is faced by education policy makers in Indonesia where 300 different languages are spoken. There, government policy is for the national language, *Bahasa Indonesia*, to be the language of instruction (Suprapto Brotosiswojo, 1995).

4. Rejecting Cultural Imperialism in Mathematics Education Policies and Practice

What we have said of language can also be said, even more generally, of "culture." Bishop (1990) has argued that although many cultures have contributed to the development of the internationalised version of "Mathematics," nevertheless "Western Mathematics" has been one of the most powerful weapons used in the

imposition of Western culture on "developing" nations by colonising powers.

According to Bishop, the reason why Western Mathematics has been so powerful is that everyone has regarded it as neutral, as the least culturally-laden of all the school subjects imposed on indigenous pupils. The mystique and values of Western Mathematics have readily been accepted by most local, indigenous leaders, for there is a common belief that, somehow, economic and educational development is to be associated with these values.

Colonialism is an attitude of mind—accepted by both the leaders and representatives of the colonising power and by those who are being colonised—that what goes on "at home" should also take place in the colonies (Clements, Grimison, & Ellerton, 1989). This "acceptance" is sometimes a conscious act, but more often it is unconscious—people behave in a colonialist way simply because that is the way they have learnt to behave (Clements et al., 1989). D'Ambrosio (1985), in concluding a paper in which he presented a case for a radical revision of mathematics curricula in Third World countries, stated that "we should not forget that colonialism grew together in a symbiotic relationship with modern science, in particular with mathematics and technology" (p. 47).

History provides a measure of support for D'Ambrosio's position. The nineteenth and twentieth centuries provided many examples of indigenous peoples uncritically accepting Western forms of mathematics as an essential component of the curricula of community schools created by their colonial supervisors. Soon after white settlement in Australia, for example, the Colonial Office established schools for "civilising" native Aboriginal children (Clements et al., 1989). Arithmetic was an important subject, and Arithmetic textbooks from England and Ireland were used. Instruction was in the English language.

Similar stories could be told across the world. Almost invariably, indigenous children struggled to cope with "foreign" subjects, especially Arithmetic. A few succeeded, but most became sacrificial lambs. When, finally, independence came, the new class of indigenous rulers were often among those who had succeeded at school—and therefore they chose to continue the emphasis on Western Mathematics in their schools. In many countries they were supported in their efforts to do this by expert foreign consultants, funded through agencies such as UNESCO, the World Bank, and the Asian Development Bank (Kitchen, 1995).

Our fourth proposition is:

> *Proposition 4:* The assumption that it is reasonable to accept a form of mathematics education which results in a large proportion of school children learning to feel incompetent and helpless so far as "Western" Mathematics is concerned, should be rejected. Alternative forms of mathematics education, by which greater value would be accorded to the cultural and linguistic backgrounds of learners, should be explored.

In putting forward this proposition we recognise that mathematics education must always be political—in the sense that it inevitably imposes certain values on students and compels them to be involved in certain activities (Harris, 1991; Kaleva, 1995; Mellin-Olsen, 1987).

After pointing out that Karl Marx and Charles Darwin were contemporaries, D'Ambrosio (1985) went on to argue that "Pure Mathematics as opposed to

Mathematics came into consideration at about the same time [as Marx and Darwin] with obvious political and philosophical overtones" (p. 47). D'Ambrosio maintained that the distinction between Pure and Applied Mathematics is highly artificial and ideologically dangerous. "Clearly," he wrote, "to revise curriculum and research priorities in such a way as to incorporate national development priorities into the scholarly practices which characterise university research is a most difficult thing to do" (p. 47)

Undoubtedly, from a political perspective, the task of developing and implementing indigenous mathematics curricula along lines suggested by D'Ambrosio, will be extraordinarily difficult. However, the time would appear to have come when governments and mathematics educators around the world must face the challenge (D'Ambrosio, 1994; Usiskin, 1994a). However, despite much rhetoric about the need for "community oriented and culturally sensitive" curricula, mathematics curricula typically emphasise "functional numeracy" which is interpreted in terms of knowing how to carry out standard numerical algorithms on whole numbers.

Kaleva (1995), writing on the situation in Papua New Guinea, has stated: "There seems to be a perception that mathematics cannot be associated with culture and if there is any association it is at the elementary level." He went on to say that "at the secondary level there are almost no examples of mathematics from [PNG] culture in the secondary school text books, although many opportunities exist for texts to show examples from culture" (p. 146).

Those responsible for perpetuating colonialist tendencies in education can always find "good" reasons to support their policies. For example, anyone wishing to introduce an outcomes-based form of education along the lines of the recent national curriculum experiment in the United Kingdom might refer to the need to reclaim traditional values in education by returning to "traditional methods of teaching and testing" (Black, 1994, p. 194). They might stress the need to strengthen the nation's schools by introducing standards in the form a system of common student outcome statements and an approach to assessment which respects and strengthens "pupils' learning and the teachers' professional role, while at the same time supporting tough and valid assessment to satisfy the legitimate demands of public accountability" (Black, 1994, p. 196).

Kellaghan and Madaus (1995) have argued persuasively that it usually makes little sense to try to transplant one education system to another when there are large within- and between-nation differences in populations, cultures, languages, values placed on education, and in the availability of resources. But during the 1980s and 1990s there have been numerous examples of nations moving to introduce modified versions of education systems which were developed in other nations. Usually these "education system" transplants have been put into place by politicians and education bureaucrats without full and proper consultation with local educators. Invariably these new transplanted programs have met with only limited success in their new environments.

5. Working Out the Implications of Situated Cognition Research Findings for Mathematics Education

Our fifth proposition arises from recent "situated cognition" research in

mathematics education. Researchers such as Carraher (1988), Lave (1988), and Saxe (1988), have found that people in all classes and walks of life are capable of performing quite complex "mathematical" operations, despite the fact that many of them are not able to perform apparently similar operations under more formal circumstances—such as when asked to do so on pencil-and-paper mathematics tests. In Australia, for example, children who could easily calculate, mentally, how much change they should receive from $5.00 if they were to buy a chocolate bar for 45 cents, were unable to find the value of 500 – 45, when this was presented as a pencil-and-paper subtraction task (Lovitt & Clarke, 1988).

But the educational implications of this research are not at all clear (Anderson, Reder, & Simon, 1996; Nunes, 1993). Although Nunes (1993) has written directly on how street mathematics can be, and has been, brought into the classroom, she also acknowledged that it was difficult to do this successfully (p. 35).

Suppose, for instance, there were three street "candy sellers" in a mathematics class of 35 junior secondary students in a school in Brazil. How can the ordinary teacher be expected to know the special abilities of these three children? After all, the teacher teaches quite a few different classes. And, even if the teacher did know of the special calculational abilities of the three children, how should that influence the teacher's normal class planning and teaching methods? There are 32 other students in the class, and no doubt many of these also have special situated mathematical knowledge (relating, for example, to sporting interests, gambling practices, and income-earning work done away from school).

Intuitively, mathematics educators are inclined to believe that situated cognition research should send important messages to mathematics teachers and mathematics curriculum developers (Nunes, 1993). There is some evidence that this is indeed the case (Noss & Hoyles, 1996; Vergnaud, 1982). But at present we do not know what these messages are and, if and when the messages have been decoded, how they can be used to maximise the learning and appreciation of mathematics. Future research needs to be directed more at exploring classroom implications of what is now a well-known phenomenon.

> *Proposition 5*: The implications of situated cognition research for mathematics curricula, and for the teaching and learning of school mathematics, needs to be investigated in creative ways.

With respect to *Proposition 5*, we do not think that controlled experiments, in which artificial education environments are created, would be helpful. The eclectic, innovative, and more naturalistic approach to mathematics education research adopted by Noss and Hoyles (1996) should be carefully considered.

6. Reconceptualising the Role of Theory in Mathematics Education Research

In Chapter 28 of *The International Handbook of Mathematics Education* (Bishop, Clements, Keitel, Kilpatrick & Laborde, 1996), Mason and Waywood (1996) identified three broad "traditions" of research practice in mathematics education. They called these traditions "pragmatic empiricism," "philosophical well-foundedness," and "results-focused," and argued that they not only represented distinguishable responses to concerns about theoretical underpinnings of research activity, but were also useful for raising awareness of other possibilities. They noted that the strong

correspondence of their traditions with the following three principle research traditions described by Bishop (1992):

• *Pedagogue* tradition, for which the goal is to improve teaching. Evidence is produced in the form of exemplary children's behaviour, and theory comprises the accumulated shareable wisdom of expert teachers.

• *Scholastic-philosopher* tradition, for which the goal is a rigorously argued theoretical position. Evidence is presumed to be already collected, and theory focuses on idealised situations to which educational reality should aim.

• *Empirical scientist* tradition, for which the goal is to explain educational reality by offering facts and measurements to be explained. Explanatory theory is generated which can be tested against past, present, and future data.

Despite the lack of agreement on what, precisely, constitutes a theory in the domain of mathematics education, leading mathematics education researchers with much influence on the international stage have often debated the role of theory in mathematics education research. Although those participating in the debate tended to talk at cross purposes—for often they came from different traditions—many of them agreed on one thing: that mathematics education research needed to become more theory-driven. Those who identified with this point of view tended to look to the spectacular achievements of theory-driven research in the physical sciences, and to argue that the quality and usefulness of mathematics education research would not improve unless a similar approach was adopted by mathematics educators. They also suggested that strong theories were needed to guide choice of research questions and methodologies, and to provide researchers with common language and ways of thinking.

Those mathematics education researchers who have preferred to work from, and extend, existing theories have tended to justify this approach to research by referring to metaphors (such as workers "building a wall," by continually adding new "bricks," the final aim being to complete the "wall" under construction).

On the other hand, many current education researchers believe that attempts, over the period 1950-1980, to develop education theory by the application of rigorous experimental designs and careful statistical analyses, or comprehensive interpretive studies, were unproductive and misguided. As (Bessant and Holbrook (1995) have stated, the results obtained "were seldom ever nationally applicable (let alone internationally)—people, cultures, regions were different" (p. 234).

The preference for theory-driven research went so far in the United States in the 1980s that some funding bodies instructed assessors of proposals to deny funding to applicants for grants who had not elaborated well-defined theoretical bases for their proposed research. Research proposals were expected to put forward models—however tentative—relating phenomena to be investigated (Romberg, 1992).

The role of theory in education research is not something which all mathematics education researchers have agreed upon, however. From our perspective, many of the theories and metaphors used to support the theories, have received more attention than they deserve. Furthermore, because some theories have become so widely accepted and used that they have come to be regarded as

being objectively "true," the overall effect of theory-driven research has not necessarily been a positive one.[62] Although we would not wish to criticise researchers who work from a given theoretical stance—based on, for example, Piaget or Vygotsky—we would defend the approach taken by researchers who are investigating an important aspect of mathematics and wish to depart from existing theories—in other words, they are exploring relationships, perhaps with a view to generating new theoretical perspectives.[63]

Certainly, the notion that global theories might apply to the field of education was attractive. It would have been useful, for example, if we could have identifed a single "best" way to introduce early number concepts to young children. However, in the 1990s such an idea would be regarded by most mathematics education researchers as naive.

We wish to assert that the domain of mathematics education is not controlled by culture-free laws which can be progressively identified through research. Mathematics educators should emphatically reject descriptions of mathematics education as being at some point in a sequence from "myths," to "traditions," to "model building," to "paradigm selection," to "scientific revolution" (Romberg, 1981, 1983). As Walker (1977, quoted in Kilpatrick, 1992), conjectured, education may not be capable of becoming a science, and because of the attempt to achieve what is not possible, educational research has been "plagued by uncertainty about its nature, fails to make continuous progress because of faddishness, and suffers from low prestige." Walker went on to say that "the fact of low prestige has led to a compulsive quest for rigor through unassailable methodology that has virtually sterilised research in the field." This has led "to a servile imitation of and dependence on established disciplines." Walker claimed that the search for academic respect had led researchers away from educational practitioners.

We are among those mathematics educators who believe there has been an over-rapid generalisation of generic education theories to the domain of mathematics education. Unlike many other researchers, we do not believe that theoretical positions, such as those represented by the Piagetian stages, by the van Hieles and SOLO Taxonomy levels, and by certain complex models for explaining rational number concept development in children, have contributed to a significant

62. Consider, for example, Piagetian stage theory, which is based on a biological metaphor. See Zevenbergen(1993) for a critique of the application of Piagetian theories in mathematics education research. Public statements made at the 1996 annual conference of the International Group for the Psychology of Mathematics Education (held in Valencia, Spain) revealed that there are still many leading members of the international mathematics education research community who believe it is presumptuous of mathematics educators to question the methods and theories advocated by Piaget and the Genevan School. We wonder why that should be the case. We, personally, do not accept, for example, Piaget's equilibration theory, and would argue that the widescale acceptance of the associated cognitive dissonance approaches have held back progress, both in theory and in pedagogical practices.

63. Thus, for example, we applaud the approach of Fong (1995), of Singapore, who, on finding that theoretical positions associated with Bloom's *Taxonomy*, Piagetian and neo-Piagetian theories of cognitive development, and the SOLO Taxonomy, were not useful in providing a rigorous explanation of mathematical problem-solving, used local data to generate an "information processing taxonomy for assessing mathematical abilities."

improvement in the quality of the teaching and learning of mathematics over the past two decades. However, powerful figures—who have insisted that research based on such theoretical models is superior to research in which more exploratory approaches are preferred—have succeeded in creating a mindset within an influential section of the international mathematics education research community.

The view that mathematics education research should resemble scientific research has resulted in theoretical structures being conjectured and much data being generated which confirm or partly confirm proposed theories. Thus, for example, Riley, Greeno and Heller's (1983) information processing model for explaining how children process "change," "combine," and "compare" addition and subtraction word problems has been enormously influential. Yet Riley et al.'s (1983) study which supported the theoretical model involved small, fairly select samples, as have most subsequent investigations based on the same theory. Furthermore, it is likely that the theory cannot be easily generalised to settings where English is not the language of instruction, because the "change," "combine," and "compare" categories which have been developed with respect to the English language may not be meaningful in other languages.

Often, seemingly attractive theories which have been developed specifically within the domain of mathematics education have quickly become set in stone, resulting in a situation where counter results have been ignored. Thus, for example, although Del Campo and Clements (1987), and Lean, Clements, and Del Campo (1990), produced data which demonstrated beyond reasonable doubt that the Riley et al. (1983) theory did not apply to Australian and Papua New Guinean children, this does not seem to have had any effect on subsequent writings of some North American researchers (Fuson, 1992; Hiebert & Carpenter, 1992).

According to Secada (1988), theories purporting to explain the development of addition and subtraction are "coherent but incomplete" (p. 33). They have been based on research which attended to majority cases. By excluding minority children from its findings, such research legitimises an ongoing view of such children as deviant.

We would go even further than Secada on this matter, by asking: Who is "normal"? And to which populations can one generalise from a study which, for instance, involved mainly middle-class Anglo-Saxon children, somewhere in a university town in an affluent Western nation?

> *Proposition 6*: The idea that the best mathematics education research is that which is based on a coherent theoretical framework should be subjected to careful scrutiny. Furthermore, popular existing theories for which strong counter data have been reported, should either be abandoned immediately, or substantially modified.

We are *not* saying that the development and application of logical classification schemes, such as the "Change, Combine, Compare" system (for additive and subtractive arithmetic word problems), and the Newman interview approach, have not had beneficial effects. We *are* saying that research rigidly based on generalisations from speculative theories that do not stand the test of having been triangulated by independent researchers working in a range of cultural and [illegible]guistic situations, is dangerously limiting.

Some of the most sensible words on this subject, it seems to us, have been written recently by David Wheeler (1996):

> Contributions to mathematics education can't be entirely theory-free any more than they can be value-free, but if mathematics education is to become a science it must give attention to producing techniques and tools that anyone can pick up and use. The histories of other sciences suggest that the adventurous unfettered thinking and committed rigorous experimentation are the prerequisites for, not the consequences of, systematic theory-building. (p. 344)

But common sense like this is often set against rhetorical statements, such as Kurt Lewin's "there is nothing as practical as good theory and nothing as theoretical as good practice."

From our perspective, at the present time mathematics education needs *less* theory-driven research, and *more* reflective, more culture-sensitive, and more *practice-oriented* research which will assist in the generation of more domain-specific theory. Although we have some empathy with the second part of Lewin's statement ("... there is nothing as theoretical as good practice"), we still regard it as somewhat of a slogan. If a good teacher believes that he/she is *not* teaching according to any particular theory, then it is patronising for someone else to say that in fact this belief is incorrect. With respect to the first part of Lewin's statement, we would wish to put forward an alternative. Our alternative statement would begin: "In mathematics education there is nothing so dangerous as speculative theory based on data deriving from a single culture."

7. *A New Epistemological Framework for Mathematics Education Research*

Our seventh proposition concerns the need to establish an epistemological framework for mathematics education research.

We believe that mathematics education research will be most informative if attention is given to bringing together the literatures on the following four key areas: (a) the histories of mathematics and mathematics education; (b) mathematical understandings and achievements in different cultures; (c) influences of culture on young (pre-school) children's conceptions of mathematics; and (d) the impact of schooling on learners' conceptions of mathematics.

An example of the type of research program we are contemplating was provided by Clements and Del Campo (1990) who, in attempting to answer the question "How natural is fraction knowledge?", provided commentaries on three bodies of literature which had a bearing on this question. These were: (a) the history of the development of fraction concepts; (b) what is known about the fraction knowledge of cultures outside the dominant European Western tradition; and (c) what is known about how very young children seem to acquire fraction concepts, and how this is influenced by culture. Another paper by the same authors (Clements & Del Campo, 1989) explored how these early rational number concepts could be influenced by schooling.

We feel it would be constructive to apply this approach to every significant theme in mathematics.

Proposition 7: A suitable framework for achieving a more unified and systematic approach to mathematics education research is needed. One possible approach would focus on research programs which linked (a) the histories of mathematics and mathematics education; (b) mathematical understandings and achievements in different cultures; (c) the influences of culture on young (pre-school) children's conceptions of mathematics; and (d) the impact of schooling on learners' conceptions of mathematics.

A framework based on *Proposition 7* would be likely to generate a more balanced and less colonialist agenda for the emerging body of mathematics education researchers in Asia-Pacific nations.[64]

8. Questioning the Basis for Assessing Achievement in Mathematics

We believe that not only are many of the instruments which are most widely used to assess mathematics achievement in research studies out-of-date, but also recent research suggests that claims made by psychometricians for their validity and reliability—of the kind found in test manuals sold by testing agencies—are based on narrow, largely statistical interpretations of these elusive concepts.

The Fall 1996 edition (Vol. 66, No. 3) of the *Harvard Educational Review* carried an article entitled "Assessment at a Crossroads: Conversations" in which 12 educators, from a range of education settings, discussed issues to do with assessment. They all agreed that fundamental changes are needed in schools so far as assessment is concerned. The National Council of Teachers of Mathematics (1995) also accepts the need for change, having published its *Assessment Standards* document in 1995 (see Joyner, 1995) and other major statements which urge teachers, schools, and school systems to play their part in reducing the current emphasis on standardised pencil-and-paper testing (see, for example, Lambdin, Kehle, & Preston, 1996). Yet, in schools in all countries, traditional pencil-and-paper tests, often of the short-answer or multiple-choice variety, continue to be the most widely used instruments for assessing mathematical learning (see, for example, Schmidt & Brosnan, 1996).

Despite research evidence pointing unambiguously to the harmful effects of high-stakes pencil-and-paper testing regimes (McLaughlin, 1995; Smith &

64. At the 8th International Congress on Mathematical Education held in Seville (Spain) in July 1996 (ICME 8), an early version of this chapter was discussed by members of Working Group 24, which was concerned with identifying criteria for evaluating mathematics education research. Three mathematics education researchers—Ramakrishnan Menon, Jeremy Kilpatrick and Christine Keitel—were charged with the responsibility of responding to the paper. They argued that Propositions 6 and 7 seemed to contradict each other, in the sense that although Proposition 6 seemed to be opposed to strong theoretical framework, Proposition 7 seemed to be recommending the use of a new theoretical framework. We would maintain that Proposition7 sets out a *framework* for future investigations without providing, or indeed implying, a strong theoretical orientation for mathematics education research. There is no suggestion that all mathematics education researchers should be expected to "cover" all four aspects mentioned in Proposition 7 in each and every investigation they plan and carry out.

Rottenberg, 1991), well-worn claims by supporters of testing (see, for example, Ebel, 1972) still carry much weight in all nations in the Asia-Pacific region. Most senior educators and leaders of society believe that pencil-and-paper testing (a) encourages and rewards individual effort; (b) has positively assisted the movement towards "quality" in educational systems; (c) has ensured that higher education and job placement are extended more on the basis of merit and less on the basis of social background; and (d) has enabled decisions on important issues in education to be made on the basis of solid evidence and less on the basis of prejudice and subjectivity (Bhogayata, 1995).

In particular, evaluators of mathematics education programs, and those responsible for the conduct of national and international mathematics achievement surveys and mathematics competitions, still regard achievement on pencil-and-paper tests as being fundamentally important. Many system bureaucrats and school principals have also been persuaded that, in these days when education accountability is expected, academic benchmarks can be established most easily and cheaply through such tests.

Mathematics education researchers often find it convenient to provide operational definitions of their main dependent and independent variables in terms of performance on pencil-and-paper tests (Sereda, 1993). Thus, for example, researchers comparing the effectiveness of mathematics programs in schools in the United States of America and certain Asian nations have made widespread use of pencil-and-paper tests (see, for example, Stigler & Baranes, 1988).

Yet, research reviewed in Chapter 6 of this book (Ellerton & Clements, 1995; Thongtawat, 1992) has generated data which suggest that students who correctly answer items on so-called "valid" and "reliable" pencil-and-paper mathematics tests sometimes have little or no understanding of the mathematical concepts and relationships that the tests were designed to measure. A strong case has been made for the extraordinary conclusion that statistically reliable pencil-and-paper mathematics tests are good at reliably generating *mis*information. This seems especially likely to be the case with tests consisting of multiple-choice items.

> *Proposition 8:* Closer research scrutiny needs to be given to the issue of how achievement is best measured in mathematics, and pressure should be exerted on education systems, testing authorities, mathematics competition directors and, indeed, mathematics education researchers themselves, to apply the findings of such research.

Clearly, there are many important areas of mathematics in which learning cannot be adequately measured by scores obtained on short-answer and multiple-choice pencil-and-paper tests (Collis, 1992; Kamii & Lewis, 1991). The mathematics education research community seems to be prepared to play a leading role in the reform of assessment practices and theory (see, for example, Glaser & Silver, 1994; National Council of Teachers of Mathematics, 1995; Webb & Coxford, 1993), and therefore should not be accused of hyprocrisy. There seems to be a recognition that pencil-and-paper tests do not measure mathematical understandings adequately, or accurately, and a determination to develop more adequate and balanced methods of monitoring and assessing students' thinking and progress.

However, there is still a strong temptation for researchers to employ ubiquitous, but inexpensive—and straightforward—pencil-and-paper assessment instruments in their investigations. It seems that it is one thing to advocate, publicly, the use of new assessment schedules for mathematics education, but another thing, altogether, to develop, validate, and use new methods in actual research studies.

9. Establishing Research Communities Which Value All Participants

The main focus of Chapter 5 of this book was the issue of who should be involved in mathematics education research, and in what ways. There is now a large body of literature which suggests that research problems and questions need to be located within the participants' spheres of knowledge (see, for example, Bishop & Blane, 1994; Clarke, 1994; Miller & Hunt, 1994). Theory will enter through the lenses of the participants' knowledge schema and "insofar as these schema involve connections with published theory, so will that theory play a part in shaping the research" (Bishop & Blane, 1994, p. 63).

Yet, the ideal of getting teachers involved as "equal partners" in mathematics education research is not something which has been easy to achieve. The Working Group on "Teachers as Researchers" attached to the International Group for the Psychology of Mathematics Education (PME) has met at a number of recent PME annual conferences, but at the 1996 PME annual conference it decided not to continue. A report prepared by the Group stated that members had spoken of "difficulties involved in gaining acceptance for a broad description of acceptable research definitions," of "problems involved in doing co-operative research with a teacher" and of difficulties "encountered in their project while working with teachers." One member drew attention to her "approach in working with teachers but allowing them to remain true to themselves" (Breen, 1996, p. 15).

Action research which attempts to maximise the potential contributions of all participants in a research exercise, at all stages of the exercise—including the design and reporting stages—and which aims at achieving improvement through co-operation, has not been as widely used in mathematics education as it might. Although some reports of successful action research projects in mathematics education have been published (see, for example, Ellerton, Kim, Madzniyah & Norjiah, 1996; Ellerton, Clements, & Skehan, 1989; Mukhopadhyay, 1996; Tan & Sim, 1991), there are far more published accounts of projects in which more traditional methodologies have been employed.

There still remains the problem of communicating the findings of mathematics education research studies to teachers in a way which will enable the teachers to use the results to help them improve their own practices. Intuitively, there is much to said for the notion of teachers as researchers. The quest to gain acceptance of such research will be an uphill one if senior academics continue to say, publicly, that action research is for school teachers, and can never take the place of more formal research. Bacchus (1995a) has shown that at least one University—Aga Khan University in Pakistan—is prepared to accept joint submissions of research theses from participants in action research projects. But this would not be possible most universities.

Desforges and Cockburn (1987) have provided supporting data for their contention that most teachers will simply not respond to the admonitions and theories of remote, self-designated "expert" researchers in mathematics education. Action research not only provides a means by which the wisdom of practice can be incorporated generously into the research, but it also maximises the likelihood that teachers will notice, and act upon, the research findings.

> *Proposition 9:* Practising teachers need to be involved, as equal partners, in mathematics education research projects, and the theoretical assumptions and practical approaches in such projects should not be predetermined by outside "experts."

Many teachers are simply unable to find the extra hours necessary to read dense education research reports, no matter how relevant these reports might, in fact, be for what they are doing. But, when the teachers close the doors to their classrooms and are confronted by an expectant class of enthusiastic faces, they are willing to try new ideas and methods that they think will help to improve their own teaching and their students' learning (Foong, 1995; Warner, 1996). That is why research methods need to be employed which result in teachers feeling a degree of ownership over the results obtained (Clarke & Clarke, 1996).

10. Making the International Mathematics Education Research Community Truly International

There is a need to end the Eurocentric/North American domination of mathematics education research which has emerged over the past three decades. For example, most of the keynote speakers at the annual conferences of the International Group for the Psychology of Mathematics Education Research (PME) have been from North America or Continental Europe, and only rarely have more than a handful of participants from China, India, Indonesia, Vietnam, the Philippines and other nations of Southeast Asia attended. Yet these last-named Asian nations contain half the world's population. It is simply not good enough to say that PME conferences are open to whoever wishes to attend—often those interested in mathematics education in these countries are not aware of international mathematics education conferences, and even if they are, are unable to find the resources needed to go.

Many researchers in these countries would be pleased to become increasingly involved in international education research communities, but without adequate support this is problematic for them (see, for example, Dang ung Van, 1996).

> *Proposition 10:* The present international mathematics education research community needs to move proactively so that full and equal participation is possible for mathematics educators in countries which are currently under-represented in the community.

Among the issues which should be taken into account are the need: (a) to identify funding sources which will assist mathematics educators from all countries not only to attend, but also to participate actively, and on a regular basis,

in important mathematics education conferences (UNESCO, the World Bank, and the Asian Development Bank may be able to provide support); (b) to foster quality research by teams of researchers containing a judicious mix of experienced and inexperienced mathematics education researchers from different countries; (c) to develop agreements by which the reporting of this research will give local researchers full credit; and (d) to expand the set of nations in which important mathematics education conferences are held.

A start has already been made to achieve greater participation of mathematics educators in countries which are currently under-represented in a number of important aspects of the international mathematics education research effort. At present, most papers published in leading mathematics education research journals, and most of the participants at major mathematics education research conferences come from Western nations. An "Under-represented Countries Group" was established in 1994 at the 18th annual conference of the International Group for the Psychology of Mathematics Education (PME-18, held in Portugal), and the Group continued to meet at PME-19 (in Brazil) and at PME-20 (in Spain). The pro-active agenda of the Group includes the following activities (Denys & Edwards, 1996):

- Identify countries under-represented at PME, consider reasons for this, and prepare a set of criteria/priorities for selecting new members;
- Compile a comprehensive list of regional contacts and individuals in countries under-represented at PME;
- Activate an e-mail discussion list, and create a WWW page for the subcommittee;
- Create a plan for the dissemination of materials and communications to countries without reliable Internet access;
- Investigate funding possibilities, including those outside of PME/ICME;
- Complete a proposal for a discussion group for PME 21;
- Discuss and plan for building research communities in home countries and for specific participation in future PMEs; and
- Investigate the meaning of the term "under-represented countries."

However, as Sitsofe Anku, originally from Africa but now working in Singapore, wrote in the November 1996 *PME Newsletter*, "current PME members will not know what they are missing unless they learn to listen to those who have been excluded (by whatever reasons) for so long, but who are making efforts to join us. The new members ... need an atmosphere of trust and encouragement by the founders of PME" (p. 8).

Anku's statement "... current PME members will not know what they are missing unless ..." warrants serious consideration.

Concluding Comments

Issues raised in this chapter need to be confronted urgently by the international mathematics education research community.

Collectively, our ten propositions emphasise the need for the international

mathematics education research community: (a) to examine fundamental assertions which currently drive research activities in mathematics education; (b) to give due accord to how linguistic and cultural factors influence mathematics education; (c) to question whether it is helpful or even reasonable to work towards the development of "grand theories," on the assumption that mathematics education is progressing towards being a science; and (d) to demonstrate a greater respect for the wisdom of practice deriving from the classroom knowledge and the action-oriented theories of practising teachers of mathematics.

Despite all the possibilities in the region, there is a sense in which the mathematics education research clock reads "five minutes to midnight." In these days of economic rationalism, the allocation of resources to education systems and to education research is being scrutinised. In many universities in Western nations, mathematics departments are struggling to maintain enrolments, and the numbers of staff in the departments are therefore being reduced.[65] The downturn has happened so suddenly, and been of such serious proportions, that Western nations are already facing daunting shortages of qualified teachers of mathematics. The situation regarding availability of mathematics teachers promises to get worse, rather than to improve, and there is a distinct possibility that in secondary schools significantly less time will have to be dedicated to the subject.

Furthermore, atomised, outcomes-based mathematics curricula and teaching approaches are being mandated by TQM-inclined education managers in many parts of the world. This small-minded trend towards reductionism threatens the relevance, and therefore the usefulness, of both mathematics and mathematics education.

But in the Asia-Pacific region "mathematics" still enjoys high status, and mathematics education researchers, freed from the forces of intellectual colonialism, need to take advantage of that status (and access to resources) to accelerate the process of reconstructing their field—before it is too late.

The governments of many of the nations in the Asia-Pacific region have set targets for government departments to achieve by some date early in the twenty-first century. We wonder what an agenda for action over the next 25 years might be like for mathematics education researchers in the Asia-Pacific region. Unless visionary and obviously relevant research is carried out and reported in the region, politicians and education bureaucrats will have no option but to implement policies which will greatly influence practice in mathematics classrooms, but which will, in fact, *not* be based on the findings of research.

Almost certainly, different mathematics education research agendas will be set for different nations in the Asia-Pacific region. This should be seen in a positive light, and should act as a stimulus for the forging of stronger regional, national and international links between mathematics education researchers. There are signs

65. On November 13, 1996, the *Australian* newspaper (p. 6) carried an article, by Guy Healy, on this theme. The article, headed "Maths decline adds to division," referred to a National Mathematics Committee survey in Australia which stated that "mathematics departments are in serious decline," and quoted the Committee chairman as saying that the survey painted a "very worrying and disturbing picture of the health of the discipline."

that this is starting to happen. For example, the Australian Research Council (1996) recommended that mathematical science researchers be "encouraged to give higher priorities than is now customary to participation in scientific conferences in South East Asia and the Pacific Rim" (p. 7). Similar recommendations should be made for mathematics education researchers, and ways of facilitating the recommendations explored proactively.

Developing and maintaining cultural sensitivities is of paramount importance (Chitoran, 1996; Ellerton, 1996). Mathematics education research in the Asia-Pacific region is in its infancy. Just as the economies of the nations in the region are flourishing, and are making their mark on the international scene, so too, mathematics education researchers in the region have the opportunity to reconstruct their field, thereby providing leadership to the rest of the world.

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